The “Gheorghe Asachi” Technical University of Iasi Romania

The „Gheorghe Asachi” Technical University of Iasi – an excellent choice for high school graduates, who decided to opt for a career in the provoking field of engineering. All our eleven faculties, endowed with laboratories and the latest equipment, where worldwide recognized specialists perform their activity, are ready to welcome and assist you in your endeavour. I am convinced that the diploma awarded by our university will become the key of success both in your career and personal life.

We look forward to welcoming you at our university!

Gouy – Stodola Theorem

Prof. Dr. Eng. Gheorghe DUMITRASCU
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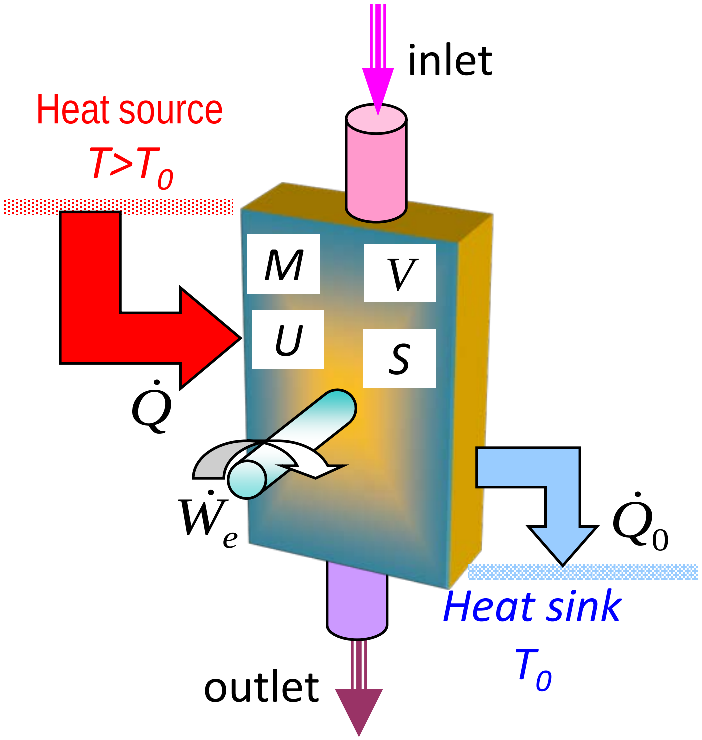
Assumptions

1. They will be analyzed non steady-state enlarged basic open thermodynamic systems, including both the thermal system and, the external heat reservoirs controlling the heat transfers and, the environment allowing the mass transfers;
2. The working fluid is a mixture of different chemical species, the inlet and outlet compositions might be different because of chemical reactions that can appear during the flow through the thermal system;
3. The inner boundary of the flow path through the thermal system is deformable under the environmental pressure;

Gouy – Stodola Theorem, Engines



First Law of Thermodynamics

$$\frac{\partial U}{\partial t} = \left(\dot{Q} - \left|\dot{Q}_0\right|\right) - \dot{W}_e - p_e \frac{\partial V}{\partial t}$$

$$+ \sum_{inlet} \dot{m}\left(h + \frac{\overline{V}^2}{2} + gZ\right) - \sum_{outlet} \dot{m}\left(h + \frac{\overline{V}^2}{2} + gZ\right)$$

Second Law of Thermodynamics

$$\dot{S}_{gen}^{irrev} = \frac{\partial S}{\partial t} - \left(\frac{\dot{Q}}{T} - \frac{\left|\dot{Q}_0\right|}{T_0}\right) - \sum_{inlet} \dot{m} \cdot s + \sum_{outlet} \dot{m} \cdot s \geq 0$$

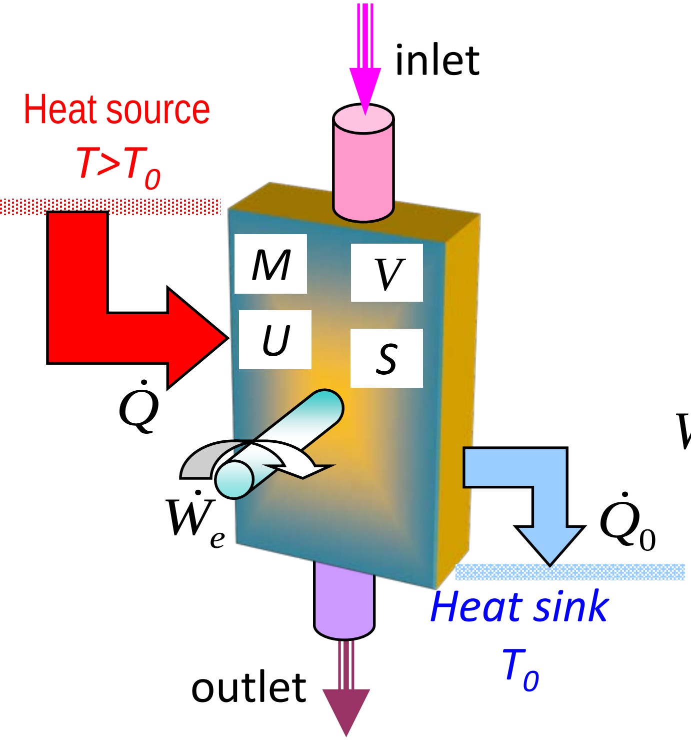
M: Mass of the working fluid surrounded by the operating engine inner walls at a certain operational time

V: Working fluid volume defined by the operating engine inner walls at a certain operational time

U: Inner operating engine working fluid energy at a certain operational time

S: Entropy of the working fluid surrounded by the operating engine inner walls at a certain operational time

Gouy – Stodola Theorem, Engines



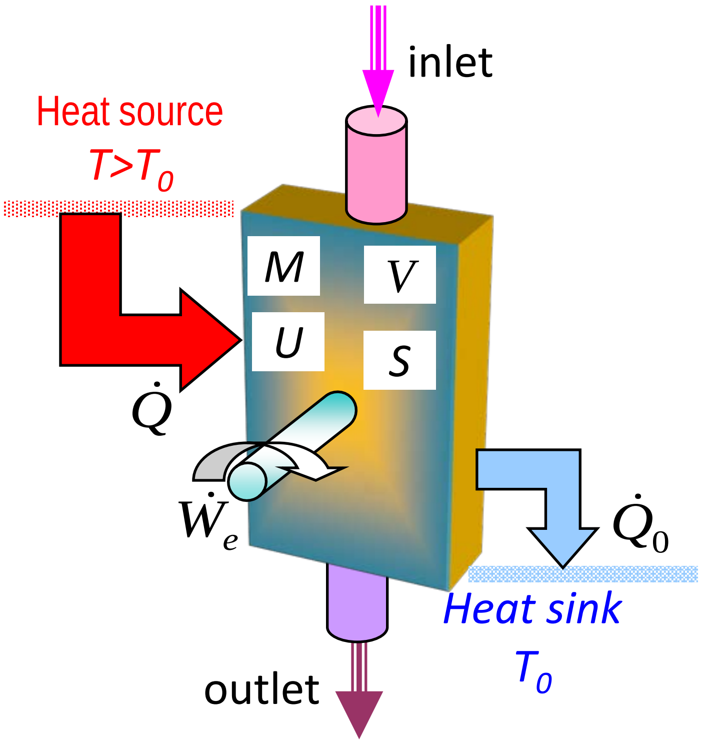
Combined First and Second Laws of Thermodynamics

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}\left(1-\frac{T_0}{T}\right)$$

$$+\sum_{inlet}\dot{m}\left((h-T_0 s)+\frac{\bar{V}^2}{2}+gZ\right)-\sum_{outlet}\dot{m}\left((h-T_0 s)+\frac{\bar{V}^2}{2}+gZ\right)$$

$$-\frac{\partial}{\partial t}\left(U+p_e V-T_0 S\right)-T_0\dot{S}_{gen}^{irrev}$$

Gouy – Stodola Theorem, Engines



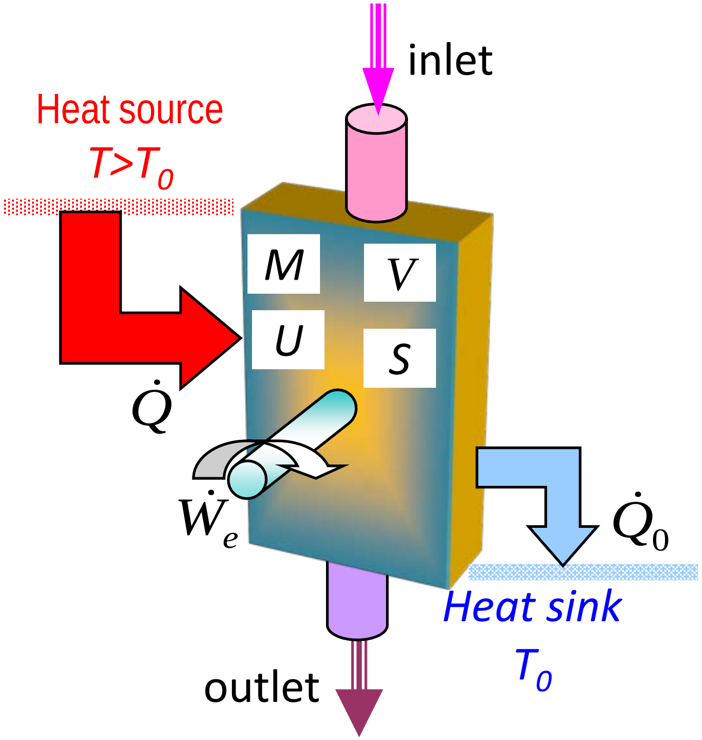
Reversible Engine Power

$$\dot{W}_e^{rev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) + \sum_{inlet} \dot{m}\left(h^* - T_0 s\right) - \sum_{outlet} \dot{m}\left(h^* - T_0 s\right) - \frac{\partial}{\partial t}\left(U + p_e V - T_0 S\right) > 0$$

where

$$h^* = h + \frac{\overline{V}^2}{2} + gZ \quad \textit{is the methalpy}$$

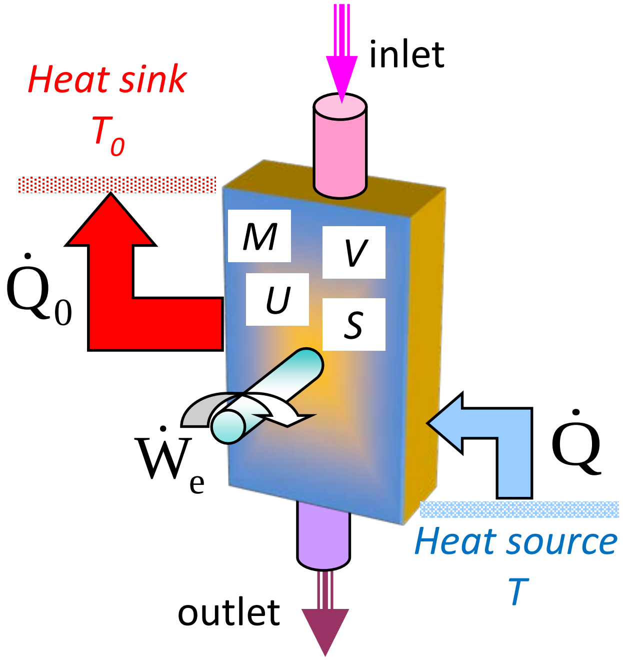
Gouy – Stodola Theorem, Engines



Irreversible Lost Power

$$\dot{W}_{lost}^{irrev} = -T_0 \cdot S_{gen}^{irrev}$$

Gouy – Stodola Theorem, Refrigeration Cycles



First Law of Thermodynamics

$$\frac{\partial U}{\partial t} = \left(\dot{Q} - \left|\dot{Q}_0\right|\right) - \dot{W}_e - p_e \frac{\partial V}{\partial t} + \sum_{inlet} \dot{m}\left(h + \frac{\overline{V}^2}{2} + gZ\right) - \sum_{outlet} \dot{m}\left(h + \frac{\overline{V}^2}{2} + gZ\right)$$

Second Law of Thermodynamics

$$\dot{S}_{gen}^{irrev} = \frac{\partial S}{\partial t} - \left(\frac{\dot{Q}}{T} - \frac{\left|\dot{Q}_0\right|}{T_0}\right) - \sum_{inlet} \dot{m}\cdot s + \sum_{outlet} \dot{m}\cdot s \geq 0$$

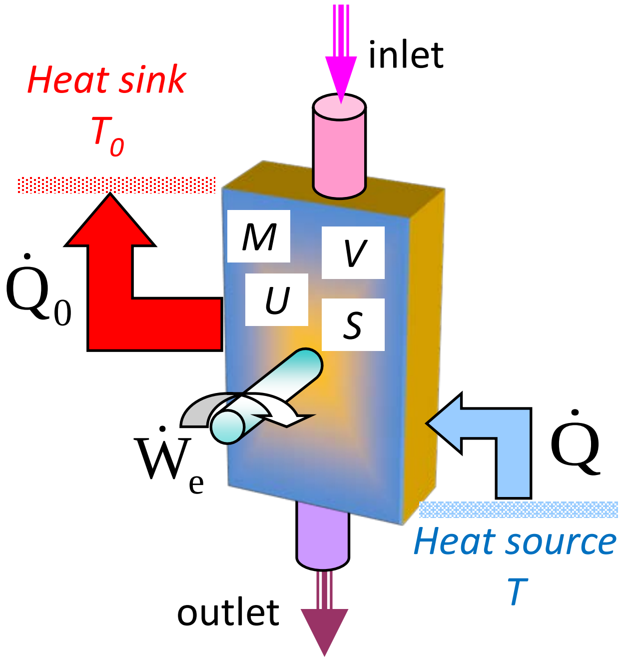
M: Mass of the working fluid surrounded by the operating engine inner walls at a certain operational time

V: Working fluid volume defined by the operating engine inner walls at a certain operational time

U: Inner operating engine working fluid energy at a certain operational time

S: Entropy of the working fluid surrounded by the operating engine inner walls at a certain operational time

Gouy – Stodola Theorem, Refrigeration Cycles



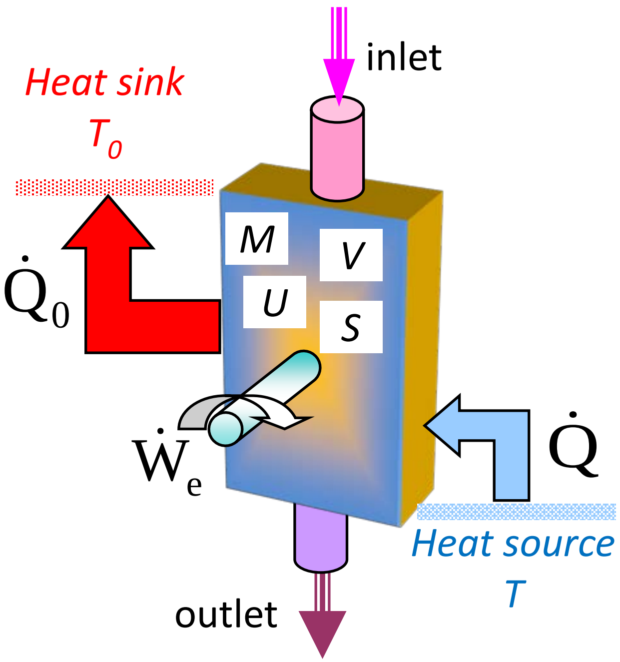
Combined First and Second Laws of Thermodynamics

$$\dot{W}_e = \dot{W}_e^{rev} + \dot{W}_{lost}^{irrev} = \dot{Q}\left(1 - \frac{T_0}{T}\right)$$

$$+\sum_{inlet} \dot{m}\left((h - T_0 s) + \frac{\overline{V}^2}{2} + gZ\right) - \sum_{outlet} \dot{m}\left((h - T_0 s) + \frac{\overline{V}^2}{2} + gZ\right)$$

$$-\frac{\partial}{\partial t}\left(U + p_e V - T_0 S\right) - T_0 \dot{S}_{gen}^{irrev}$$

Gouy – Stodola Theorem, Refrigeration Cycles



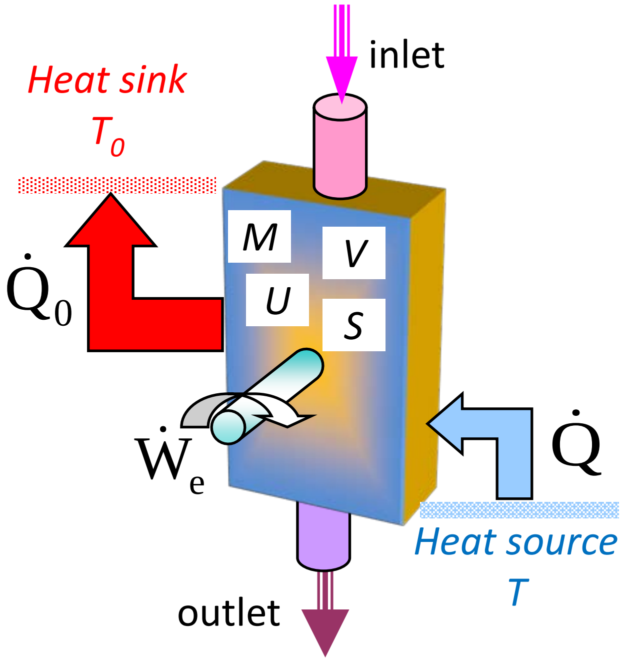
Reversible Consumed Power

$$\dot{W}_e^{rev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) + \sum_{inlet} \dot{m}\left(h^* - T_0 s\right) - \sum_{outlet} \dot{m}\left(h^* - T_0 s\right) - \frac{\partial}{\partial t}\left(U + p_e V - T_0 S\right) < 0$$

where

$$h^* = h + \frac{\overline{V}^2}{2} + gZ \quad \textit{is the methalpy}$$

Gouy – Stodola Theorem, Refrigeration Cycles



Irreversible Lost Power

$$\dot{W}_{lost}^{irrev} = -T_0 \cdot S_{gen}^{irrev}$$

Gouy – Stodola Theorem
Conclusions

$$\dot{S}_{gen}^{irrev} \to 0 \quad \dot{W}_{lost}^{irrev} = -T_0 \cdot S_{gen}^{irrev} \to 0$$

$$\dot{W}_e^{rev} = \dot{W}_{e,\dot{Q}}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev}$$

Gouy – Stodola Theorem
Conclusions

$$\dot{W}_e^{rev} = \dot{W}_{e,\dot{Q}}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev}$$

- Relationship Q – W, engines, (heat exergy)

$$\dot{W}_{e,\dot{Q}}^{rev} = \dot{Q}\left(1 - \frac{T_0}{T}\right) > 0$$

- Relationship Q – W, refrigeration cycles, (heat exergy)

$$\dot{W}_{e,\dot{Q}}^{rev} = -\dot{Q}\left(\frac{T_0}{T} - 1\right) = -\frac{\dot{Q}}{T/(T_0 - T)} < 0$$

Gouy – Stodola Theorem
Conclusions

$$\dot{W}_e^{rev} = \dot{W}_{e,\dot{Q}}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev}$$

Relationship flow– W, (flow exergy)

$$\dot{W}_{e,flow}^{rev} = \sum_{inlet} \dot{m}\left(h^* - T_0 s\right) - \sum_{outlet} \dot{m}\left(h^* - T_0 s\right)$$

Gouy – Stodola Theorem
Conclusions

$$\dot{W}_e^{rev} = \dot{W}_{e,\dot{Q}}^{rev} + \dot{W}_{e,flow}^{rev} + \dot{W}_{e,storage}^{rev}$$

Relationship “energy storage” – W, non steady-state system operation, (storage exergy)

$$\dot{\mathrm{W}}_{\mathrm{e,storage}}^{\mathrm{rev}} = -\frac{\partial}{\partial \mathrm{t}}\left(\mathrm{U} + \mathrm{p_e V} - \mathrm{T_0 S}\right)$$

Introduction to Irreversible Cycles

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Introduction to Irreversible Cycles

Assumptions

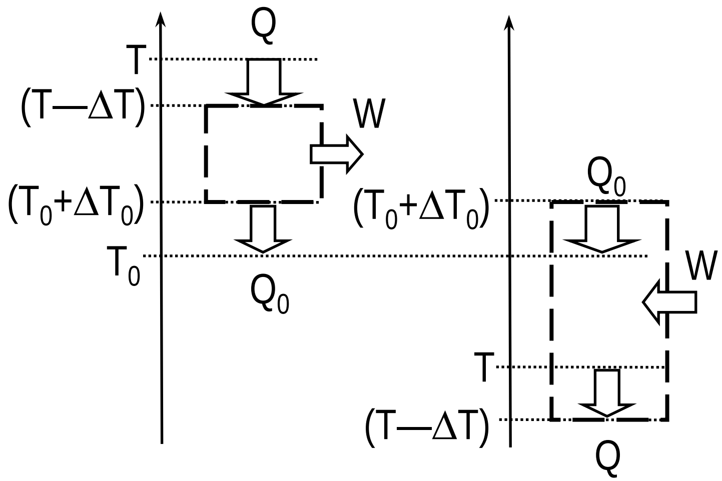
No mass transfer:

$$\sum_{inlet} \dot{m}\left(h+\frac{\overline{V}^2}{2}+gZ\right)-\sum_{\text{outlet}} \dot{m}\left(h+\frac{\overline{V}^2}{2}+gZ\right)=0 \quad \text{and} \quad -\sum_{inlet} \dot{m}\cdot s+\sum_{\text{outlet}} \dot{m}\cdot s=0$$

Non deformable boundary walls: $p_e \frac{\partial V}{\partial t}=0$

Steady state operation: $\frac{\partial U}{\partial t}=0$ and $\frac{\partial S}{\partial t}=0$

Introduction to Irreversible Closed Cycles Endo-reversible Carnot Cycle



Entropy Generation by Irreversibility

Enlarged System – including both the thermal system and the external heat reservoirs

Related to overall Irreversibility (external + internal)

$$\dot{S}_{gen}^{overall} = -\frac{|\dot{Q}|}{T} + \frac{\dot{Q}_0}{T_0} \geq 0$$

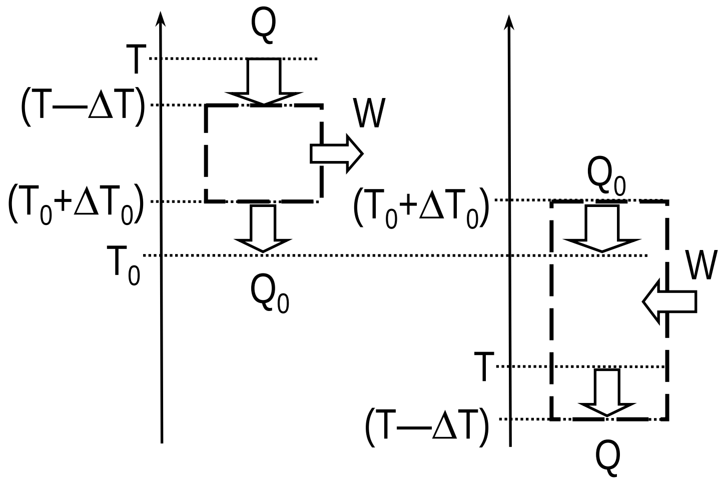
Entropy Generation by Irreversibility

Thermal System – excluding the external heat reservoirs

Related to internal irreversibility

$$\dot{S}_{gen}^{cycle} = -\frac{|\dot{Q}_0|}{T_0 + \Delta T_0} + \frac{\dot{Q}}{T - \Delta T} \geq 0$$

Introduction to Irreversible Closed Cycles
Endo-reversible Carnot cycle



Entropy Balance

Enlarged System – including both the thermal system and the external heat reservoirs

$$-\overline{\mathrm{Irr}}\frac{|\dot{Q}|}{T}+\frac{\dot{Q}_0}{T_0}=0$$

Entropy Balance

Thermal System – excluding the external heat reservoirs

Related to internal irreversibility

$$-\frac{|\dot{Q}_0|}{T_0+\Delta T_0}+N_{irrev}^{internal}\frac{\dot{Q}}{T-\Delta T}=-\frac{|\dot{Q}_0|}{T_0^*}+\mathrm{Irr}\frac{\dot{Q}}{T^*}=0$$

$\overline{\mathrm{Irr}}$ or N_{irrev} is the overall irreversibility function related to T and T_0,

$N_{irrev}^{internal}$ is the number of internal irreversibility related to (T - ΔT) and (T0 - ΔT_0),

Irr is the internal irreversibility function related to other reference temperatures on the cycle T_0^* and T^* different from (T - ΔT) and (T0 - ΔT_0)

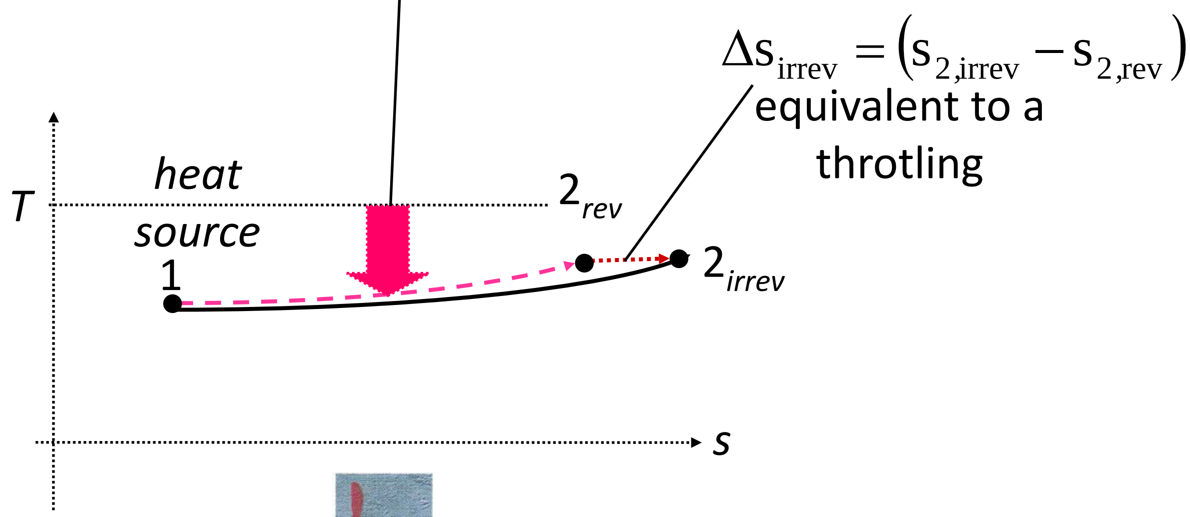
Introduction to Irreversible non-Carnot Closed Cycles

Cyclic Heat Input

$$h_{2irrev} = h_{2rev} \Rightarrow T_{2irrev} \cong T_{2rev}$$

$$\dot{Q}_{\substack{heat\\input}} = \dot{Q}_{rev} = \dot{m} T_{mq} \Delta s_q$$

$$\dot{Q}_{12irrev} = \dot{Q}_{1-2rev} = \dot{m} T_{mq}^{1-2rev} \left(s_{2,rev} - s_1 \right)$$



Introduction to Irreversible non-Carnot Closed Cycles

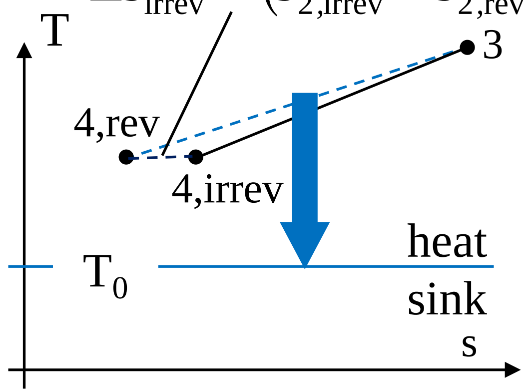
Cyclic Heat Output

$$h_{4irrev} = h_{4rev} \Rightarrow T_{4irrev} \cong T_{4rev}$$

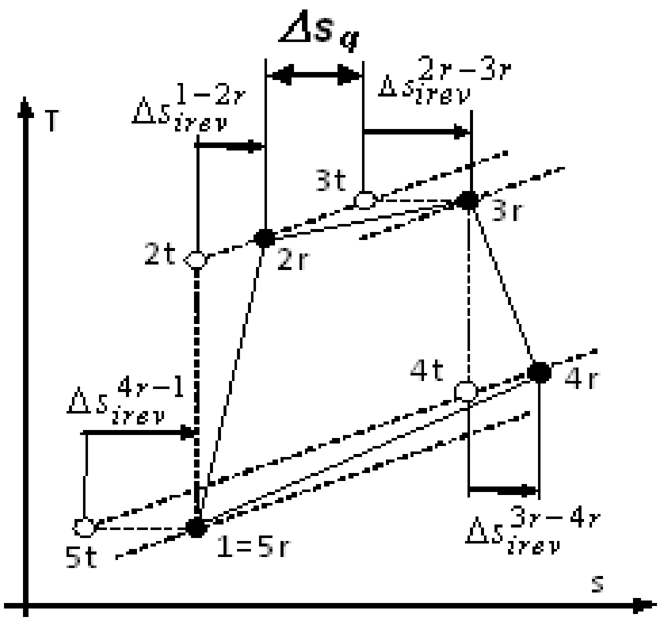
$$\dot{Q}_{\substack{heat \\ input}} = \dot{Q}_{rev} = \dot{m} T_{mq} \Delta s_q$$

$$\dot{Q}_{34irrev} = \dot{Q}_{3-4rev} = \dot{m} T_{mq}^{3-4rev} \left(s_{4,rev} - s_3 \right)$$

$$\Delta s_{irrev} = \left(s_{2,irrev} - s_{2,rev} \right)$$ equivalent to a throttling



Introduction to Irreversible non Carnot Closed Cycles



Irreversible First Law Efficiency - Engines

$$\mathrm{FLE}_{\mathrm{engines}}^{\mathrm{irrev}} = \eta_{\mathrm{irrev}} = \frac{\dot{W}_e}{\dot{Q}} = \frac{\dot{W}_e^{\mathrm{rev}} + \dot{W}_e^{\mathrm{irrev}}}{\dot{Q}_{\mathrm{rev}}} = \frac{\dot{W}_e^{\mathrm{rev}}}{\dot{Q}_{\mathrm{rev}}} - \frac{T_0 \dot{S}_{\mathrm{gen}}}{\dot{Q}_{\mathrm{rev}}}$$

$$= 1 - \frac{T_0}{T} - \frac{T_0 \dot{S}_{\mathrm{gen}}}{\dot{m} T_{\mathrm{mq}}^{2r-3t} \Delta s_q} = 1 - \frac{T_0}{T}\left(1 + \frac{\dot{S}_{\mathrm{gen}}}{\theta_{\mathrm{SLT}} \dot{m} \Delta s_q}\right)$$

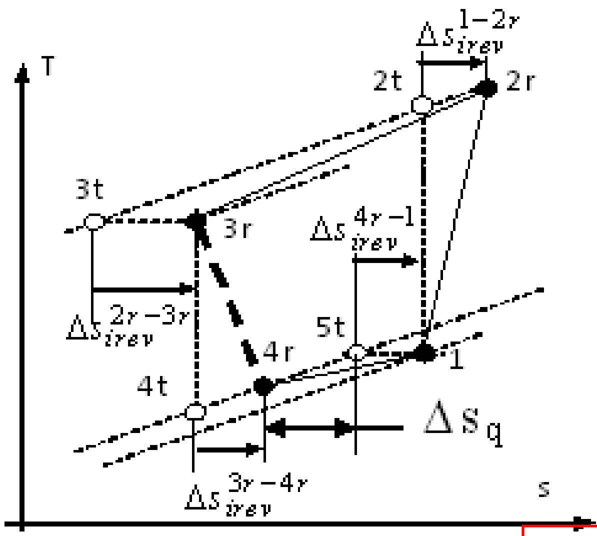
$$\eta_{\mathrm{irrev}} = 1 - \frac{T_0}{T} N_{\mathrm{irrev}} \overset{N_{\mathrm{irrev}}>1}{<} 1 - \frac{T_0}{T} = \eta_{\mathrm{Carnot}}\left(T_0, T\right)$$

Enlarged System Entropy Balance Equation

$$-\frac{\left|\dot{Q}\right|}{T} N_{\mathrm{irrev}} + \frac{\dot{Q}_0}{T_0} = 0$$

Where T is the temperature of the heat source and, $\theta_{\mathrm{SLT}} = \frac{T}{T_{\mathrm{mq}}^{2r-3t}}$ is a second law of thermodynamics correction function linking the heat transfer to the involved mean thermodynamic temperatures

Introduction to Irreversible non Carnot Closed Cycles



Irreversible First Law Efficiency - Refrigeration Cycles

$$FLE_{refrigeration}^{irrev} = COP_{irrev} = \frac{\dot{Q}}{\left|\dot{W}_e\right|} \overset{\dot{W}_e<0}{=} -\frac{\dot{Q}_{rev}}{\dot{W}_e} = -\frac{\dot{Q}_{rev}}{\dot{W}_e^{rev} + \dot{W}_{e,lost}^{irrev}}$$

$$= -\frac{1}{\frac{\dot{W}_e^{rev}}{\dot{Q}_{rev}} + \frac{\dot{W}_{e,lost}^{irrev}}{\dot{Q}_{rev}}} \overset{T<T_0}{=} -\frac{1}{1 - \frac{T_0}{T} - \frac{T_0\dot{S}_{gen}}{\dot{Q}_{rev}}}$$

$$COP_{irrev} = -\frac{1}{1 - \frac{T_0}{T} - \frac{T_0\dot{S}_{gen}}{\theta_{SLT}\dot{m}T_{mq}^{4r-5t}\Delta s_q}} = -\frac{1}{1 - \frac{T_0}{T}\left(1 + \frac{\dot{S}_{gen}}{\theta_{SLT}\dot{m}\Delta s_q}\right)}$$

$$COP_{irrev} = \frac{T}{T_0\left(1 + \frac{\dot{S}_{gen}}{\theta_{SLT}\dot{m}\Delta s_q}\right) - T} = \frac{T}{T_0 N_{irrev} - T} \overset{N_{irrev}>1}{<} \frac{T}{T_0 - T} = COP_{Carnot}\left(T, T_0\right)$$

Enlarged System Entropy Balance Equation

$$-\frac{\left|\dot{Q}\right|}{T} N_{irrev} + \frac{\dot{Q}_0}{T_0} = 0$$

Where T is the temperature of the heat source and, $\theta_{SLT} = \frac{T}{T_{mq}^{4r-5t}}$ is a second law of thermodynamics correction function linking the heat transfer to the involved mean thermodynamic temperatures

Introduction to Irreversible non Carnot Closed Cycles

Conclusions

1. η_{irrev} and COPirrev includes explicitly the overall irreversibility (internal and external one)

2. $N_{irrev} = \left(1 + \frac{\dot{S}_{gen}}{\theta_{SLT}\dot{m}\Delta s_q}\right)$ has the meaning of the overall number of irreversibility, it evaluates both the external irreversibility due to the heat transfer at finite temperature differences, and the internal ones.

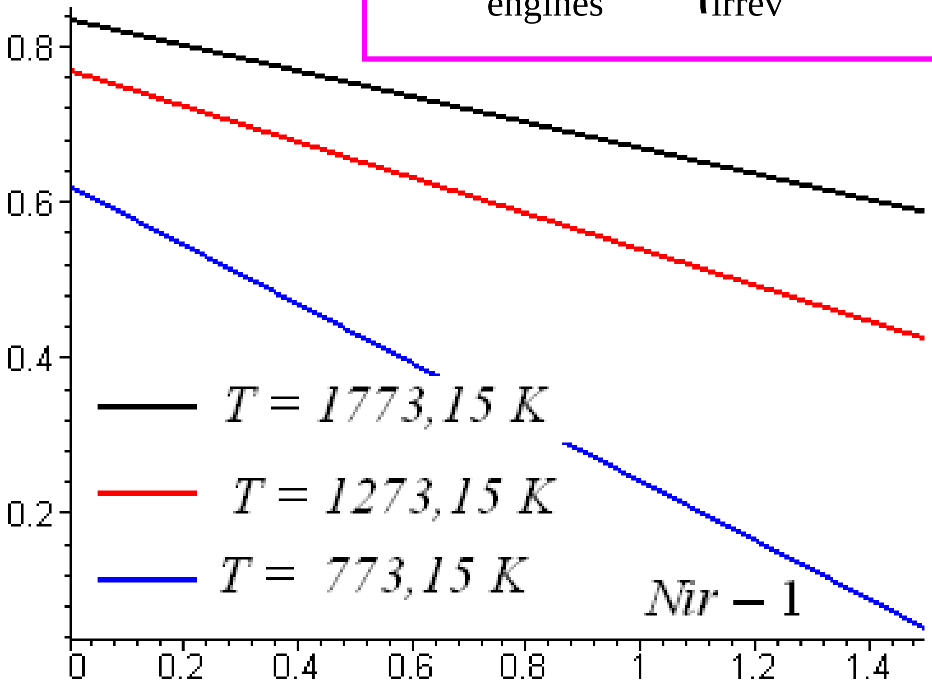
3. When $S_{gen} \rightarrow 0$ then $N_{irrev} \rightarrow 1$,

4. The correction function θ_{SLT} links the heat input at temperature T to the heat absorbed by the working fluid at temperature $T_{mq} < T$

Introduction to Irreversible non Carnot Closed Cycles

Conclusions – Engines

$$FLE^{irrev}_{engines} = \eta_{irrev} = 1 - \frac{T_0}{T} N_{irrev} \overset{N_{irrev}>1}{<} 1 - \frac{T_0}{T} = \eta_{Carnot}(T_0, T)$$

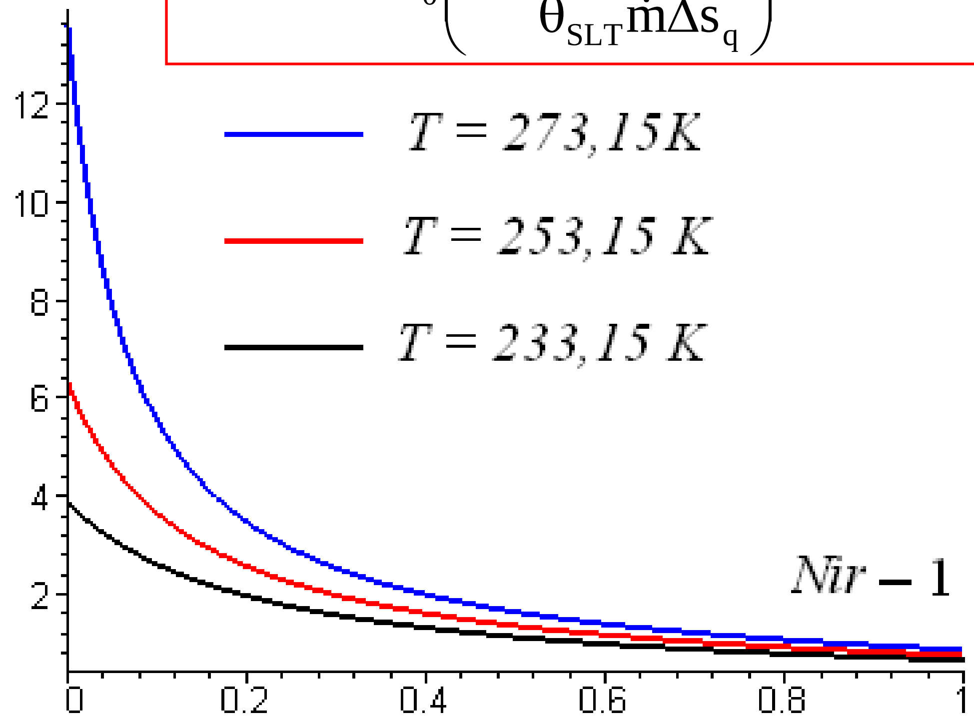


$T_0 = 273{,}15$ K

Introduction to Irreversible non Carnot Closed Cycles

Conclusions – Refrigeration cycles

$$COP_{irrev} = \frac{T}{T_0\left(1+\frac{\dot{S}_{gen}}{\theta_{SLT}\dot{m}\Delta s_q}\right)-T} = \frac{T}{T_0 N_{irrev} - T} \overset{N_{irrev}>1}{<} \frac{T}{T_0 - T} = COP_{Carnot}\left(T, T_0\right)$$



T_0 + 293,15 K

INFLUENCE OF WORKING FLUID PROPERTIES ON INTERNAL IRREVERSIBILITY OF POWER SYSTEMS

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THE WAY TO OPTIMIZE THE IRREVERSIBLE CYCLES

The optimization of cycles:

- ✓first and second law efficiencies (exergy analysis),
- ✓entropy generation minimization, and sometimes
- ✓Novikov–Curzon–Ahlborn maximum power issue.

The paper presents a concise method to evaluate directly the irreversibility, inside a unique criterion uniting first and second laws, called here ***the irreversible first law efficiency***.

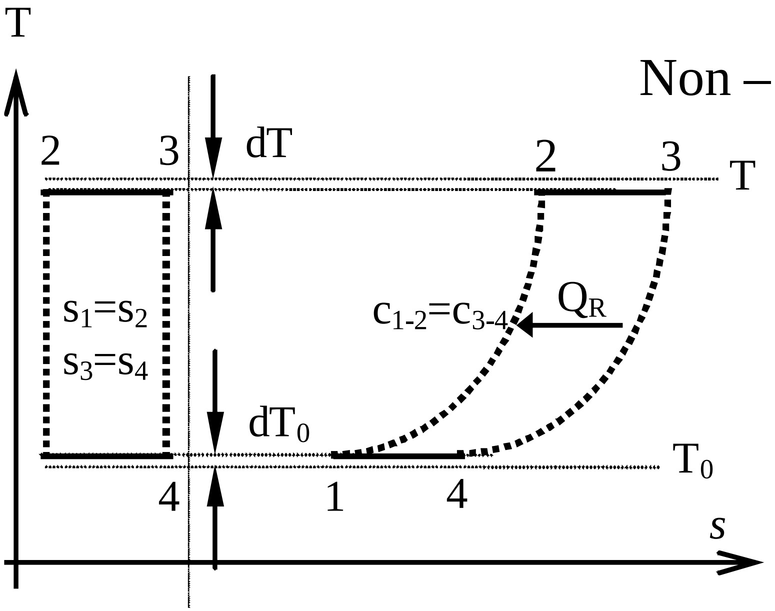
Key words:

- ✓NTUS, second law effectiveness of external heat exchanges
- ✓irreversible maximum power,
- ✓number of internal irreversibility, number of external irreversibility
- ✓irreversible first law efficiency

Engines Reversibility Principle
- engineering realm -

Any ideal cycle is characterized by no entropy generation $\dot{S}_{gen} = 0.$

Non – CARNOT complete Reversible Engine Cycles



The possible ideal engine cycles
in temperature – entropy diagram

$$s_2 - s_1 = s_3 - s_4 = c_n \ln\left(T/T_0\right)$$

All Complete Reversible Engine Cycles

$$\eta_{rev} = 1 - \frac{T_0}{T}$$

Introduction

"Optimization of Irreversible Cycles" , FTT or FST

The ideal cycles are characterized by no entropy generation, $\dot{S}_{gen} = 0.$

The ***Finite Time Thermodynamics (FTT),*** or ***Finite Speed Thermodynamics (FST)*** is a feature of the originator works of:

- CHAMBADAL (1957) and NOVIKOV (1958) – studies about nuclear cycles,
- CURZON et AHLBORN (1975) – the time-based thermodynamic analysis in view of the real heat transfer made at finite temperature difference,
- YAN et al. (1989), GROSU et al. (2004), CHEN et al. (1997, 1999) – analyzed the irreversible cycles with three external heat sinks

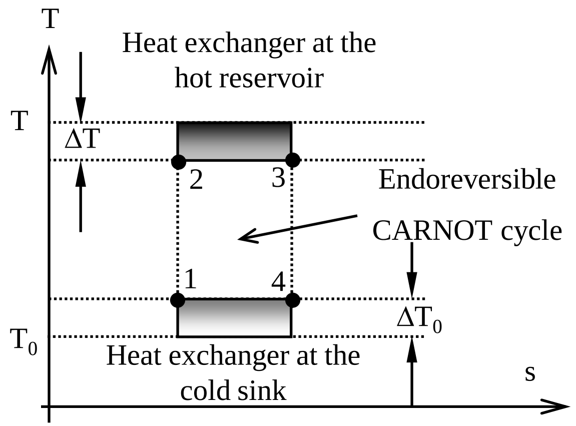
THE IDEAL REVERSIBLE CYCLES ARE CONSIDERED AS USELESS, THEY MIGHT SUPPLY THE MAXIMUM ENGINES WORK, BUT NO POWER. THE REQUIRED TIME BY THE REVERSIBLE HEAT TRANSFER AT INFINITESIMAL TEMPERATURE DIFFERENCE REQUIRES AN INFINITE PROCESS TIME (I.E. THE POWER, WHICH IS RATIO WORK PER TIME IS IN FACT ZERO).

The Irreversible First Law Efficiency

External Irreversibility

Endo-Reversible CARNOT Engine

The second law effectiveness of the heat exchange with the heat source



Number of Transfer Units per Reversible Entropy Variation due to reversible heat transfer

$$NTUS = \frac{UA}{\dot{m}\Delta s_q}$$

Basic Equations

$$\dot{Q} = UA\Delta T = \dot{m}(T - \Delta T)\Delta s_q = \dot{Q}_{rev}$$

$$\Delta T = T\frac{1}{NTUS+1} \Rightarrow \dot{Q} = \dot{m}(T-\Delta T)\Delta s_q = \dot{m}T\Delta s_q\frac{NTUS}{NTUS+1}$$

The second law effectiveness at the heat exchange at the hot reservoir

$$\varepsilon_{II} = \frac{\dot{Q}}{(\dot{Q})_{NTUS\to\infty}} = \frac{NTUS}{NTUS+1} < 1 \Rightarrow$$

$$\dot{Q} = \varepsilon_{II}\dot{Q}_{NTUS\to\infty} = \varepsilon_{II}\dot{Q}^{rev}_{max} = \varepsilon_{II}\dot{m}T\Delta s_q \Rightarrow$$

$$\Delta s_q = (\Delta s_q)_{NTUS\to\infty}, \quad \Delta T \to 0$$

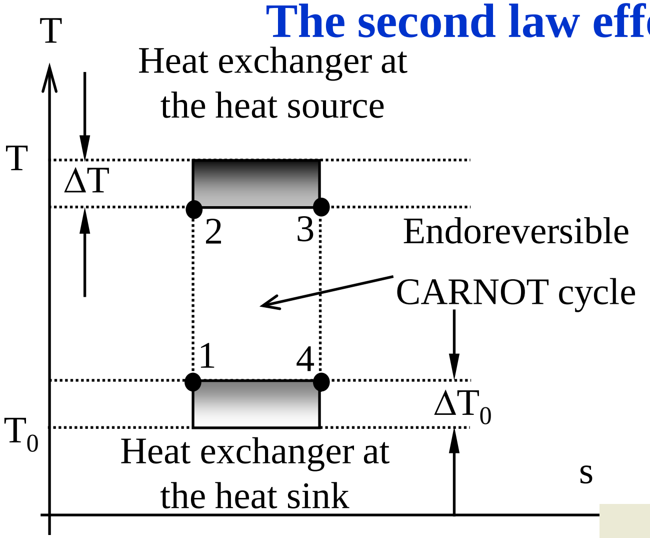
$$\dot{Q} < \dot{Q}^{rev}_{max}$$

The Irreversible First Law Efficiency

External Irreversibility

Endo-Reversible CARNOT Engine

The second law effectiveness of the heat exchange with the heat sink



Number of Transfer Units per Reversible Entropy Variation due to reversible heat transfer

$$NTUS_0 = \frac{U_0 A_0}{\dot{m}\left|\Delta s_{q,0}\right|}$$

Basic Equations

$$\left|\dot{Q}_0\right| = U_0 A_0 \Delta T_0 = \dot{m}\left(T_0 + \Delta T_0\right)\left|\Delta s_{q,0}\right|$$

$$\Delta T_0 = T_0 \frac{1}{NTUS_0 - 1} \Rightarrow \left|\dot{Q}_0\right| = \dot{m}\left(T_0 - \Delta T_0\right)\left|\Delta s_{q,0}\right| = \dot{m} T_0 \left|\Delta s_{q,0}\right| \frac{NTUS_0}{NTUS_0 - 1}$$

The second law effectiveness of the heat exchange at the cold sink

$$\varepsilon_{II,0} = \frac{\left|\dot{Q}_0\right|}{\left(\left|\dot{Q}_0\right|\right)_{NTUS_0 \to \infty}} = \frac{NTUS_0}{NTUS_0 - 1} > 1 \Rightarrow \left|\dot{Q}_0\right| = \varepsilon_0 \left(\left|\dot{Q}_0\right|\right)_{NTUS_0 \to \infty} = \varepsilon_{II,0}\left|\dot{Q}_{min}\right| = \varepsilon_{II,0}\dot{m} T_0 \left|\Delta s_{q,0}\right| \Rightarrow \left|\dot{Q}_0\right| > \left|\dot{Q}_{min}\right|$$

$$\left|\Delta s_{q,0}\right| = \left|\Delta s_{q,0}\right|_{NTUS_0 \to \infty}, \quad \Delta T_0 \to 0$$

The Irreversible First Law Efficiency

External Irreversibility

Endo-Reversible CARNOT Engine

0 | 0.5 | 1 | 1.5 | 2

$\varepsilon \to 1$

The best way

$1 \leftarrow \varepsilon_0$

The best way

$NTUS \to \infty$

Decreasing irreversibility:

$\Delta T \to 0$

$\dot{Q} \to \dot{Q}_{max} = \dot{m} T \Delta s$

$\infty \leftarrow NTUS_0$

Decreasing irreversibility:

$0 \leftarrow \Delta T_0$

$\dot{m} T_0 \Delta s = \dot{Q}_{min} \leftarrow \dot{Q}_0$

***The dependence second law effectiveness – NTU*S**

The Irreversible First Law Efficiency

External Irreversibility

Endo-Reversible CARNOT Engine

The Power

$$P = \dot{Q} - \left|\dot{Q}_0\right| = \dot{Q}\left(1 - \frac{\left|\dot{Q}_0\right|}{\dot{Q}}\right) = Q_{max}\varepsilon_{II}\left(1 - \frac{\left|\dot{Q}_{min}\right|}{\dot{Q}_{max}}\frac{\varepsilon_{II,0}}{\varepsilon_{II}}\right) = \dot{m}T\Delta s_q\varepsilon_{II}\left(1 - \frac{1}{\tau}\frac{\varepsilon_{II,0}}{\varepsilon_{II}}\right)$$

The Irreversible First Law Efficiency

$$\eta_I = 1 - \frac{1}{\tau}\frac{\varepsilon_{II,0}}{\varepsilon_{II}} = 1 - \frac{1}{\tau}N_{irr,ext}$$

The Second Law Efficiency

$$\eta_{II} = \frac{1 - \frac{1}{\tau}N_{irr,ext}}{1 - \frac{1}{\tau}} \Rightarrow N_{irr,ext} \to 1 \Rightarrow \eta_{II} = 1$$

Considerations on the Real Power Cycles

The real power cycles are also internally irreversible, respectively:

- the heat transfer processes, 2–3 and 4–1, are non-isothermal;
- the adiabatic processes, 2–3 and 4–1, are non-isentropic;
- the external heat reservoirs have finite heat capacities, respectively are non-isothermal;
- the mean log temperature difference pertaining to the heat transfers has a value that it is not equalizing the difference of the mean thermodynamic temperatures.

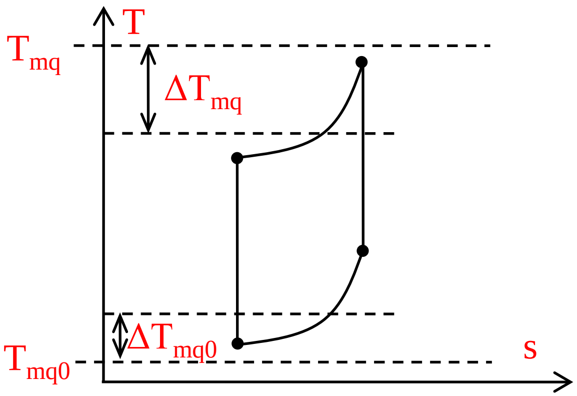
This dissimilarity can be solved step by step by introducing the necessary amendment coefficients in order to take into account also the internal irreversibility.

The Irreversible First Law Efficiency

External Irreversibility

Any Endo-Reversible Engine

The second law effectiveness of the heat exchange with the heat source



Number of Transfer Units per Entropy Variation

$$NTUS = \frac{UA}{\dot{m}\Delta s_q}$$

Basic Equations

$$\dot{Q} = UA\Delta T_{mq} C_{\Delta T} = \dot{m}\left(T_{mq} - \Delta T_{mq}\right)\Delta s_q$$

$$\Delta T_{mq} = T_{mq}\frac{1}{NTUS \cdot C_{\Delta T} + 1} \Rightarrow$$

$$\dot{Q} = \dot{m}\left(T_{mq} - \Delta T_{mq}\right)\Delta s = \dot{m}T_{mq}\Delta s_q \frac{NTUS \cdot C_{\Delta T}}{NTUS \cdot C_{\Delta T} + 1}$$

The second law effectiveness at the heat source

$$\varepsilon_{II} = \frac{\dot{Q}}{\left(\dot{Q}\right)_{\substack{NTUS\to\infty \\ A\to\infty}}} = C_{\Delta s_q}\frac{NTUS \cdot C_{\Delta T}}{NTUS \cdot C_{\Delta T} + 1} < 1 \Rightarrow$$

$$\dot{Q} = \varepsilon_{II}\dot{Q}_{max} = \varepsilon_{II}\dot{m}T_{mq}\left(\Delta s_q\right)_{\substack{NTUS\to\infty \\ A\to\infty}} \Rightarrow$$

$$C_{\Delta T} = \frac{\Delta T_{mean}^{\substack{heat \\ treansfer}}}{\Delta T_{mq}}, \quad C_{\Delta s_q} = \frac{\Delta s_q}{\left(\Delta s_q\right)_{\substack{NTUS\to\infty \\ A\to\infty}}}$$

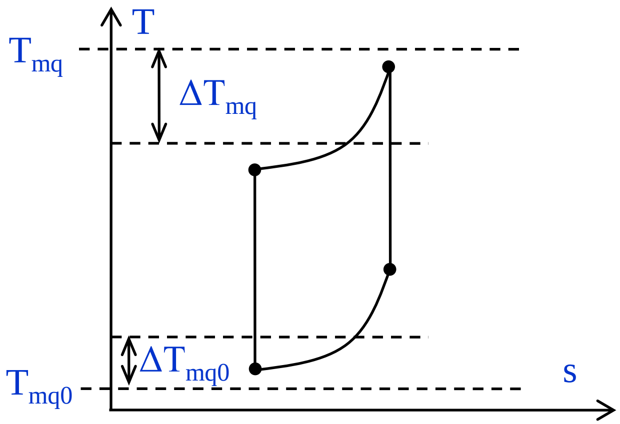
$$\dot{Q} < \dot{Q}_{max}$$

The Irreversible First Law Efficiency

External Irreversibility

Any Endo-Reversible Engine

The second law effectiveness of the heat exchange with the heat sink



Number of Transfer Units per Entropy Variation

$$NTUS_0 = \frac{U_0 A_0}{\dot{m}\Delta s_{q0}}$$

Basic Equations

$$\left|\dot{Q}_0\right| = U_0 A_0 \Delta T_{mq0} C_{\Delta T0} = \dot{m}\left(T_{mq0} + \Delta T_{mq0}\right)\left|\Delta s_{q0}\right|$$

$$\Delta T_{mq0} = T_{mq0}\frac{1}{NTUS_0 \cdot C_{\Delta T0} - 1} \Rightarrow \left|\dot{Q}_0\right| = \dot{m}\left(T_{mq0} - \Delta T_{mq0}\right)\left|\Delta s_{q0}\right| = \dot{m}T_{mq0}\left|\Delta s_{q0}\right|\frac{NTUS_0 \cdot C_{\Delta T0}}{NTUS_0 \cdot C_{\Delta T0} - 1}$$

The second law effectiveness at the heat sink

$$\varepsilon_{II,0} = \frac{\left|\dot{Q}_0\right|}{\left(\left|\dot{Q}_0\right|\right)_{\substack{NTUS_0\to\infty \\ A\to\infty}}} = C_{\Delta s_{q0}}\frac{NTUS_0 \cdot C_{\Delta T0}}{NTUS_0 \cdot C_{\Delta T0} - 1} > 1 \Rightarrow$$

$$\left|\dot{Q}_0\right| = \varepsilon_{II,0}\left|\dot{Q}_{min}\right| = \varepsilon_{II,0}\dot{m}T_{mq0}\left|\Delta s_{q0}\right|_{\substack{NTUS\to\infty \\ A\to\infty}}$$

$$C_{\Delta T0} = \frac{\Delta T_{mean}^{heat\ treansfer}}{\Delta T_{mq}}, \quad C_{\Delta s_{q0}} = \frac{\Delta s_{q0}}{\left(\Delta s_{q0}\right)_{\substack{NTUS_0\to\infty \\ A\to\infty}}}$$

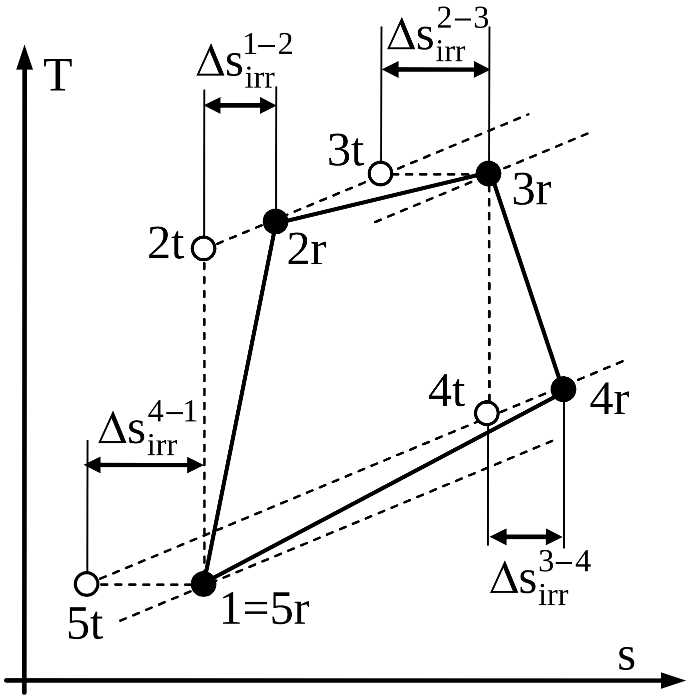
$$\Rightarrow \left|\dot{Q}_0\right| > \left|\dot{Q}_{min}\right|$$

The Irreversible First Law Efficiency

Internal Irreversibility

Any Basic Irreversible Engine

1 – 2r irreversible adiabatic compression, 2r – 3r irreversible heating
3r – 4r irreversible adiabatic expansion, 4r – 1 irreversible cooling



$$\Delta s_q = s_{3t} - s_{2r}$$

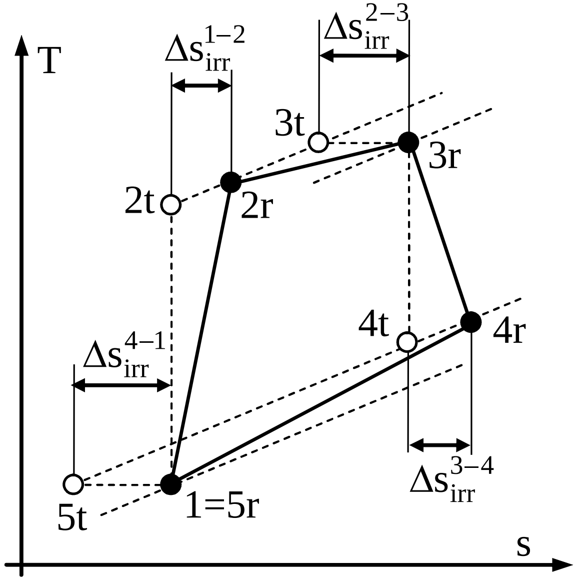
$$\left|\Delta s_{q0}\right| = s_{4r} - s_{5t}$$
$$= s_q + \Delta s_{irr}^{1-2} + \Delta s_{irr}^{2-3} + \Delta s_{irr}^{3-4} + \Delta s_{irr}^{4-1} =$$
$$= \Delta s_q \cdot N_{irr,int}$$

$$N_{irr,int} = \left(1 + \frac{\left(\sum \Delta s_{irr}\right)_{int}}{\Delta s_q}\right) > 1$$

The Irreversible First Law Efficiency

Internal Irreversibility

Any Basic Irreversible Engine



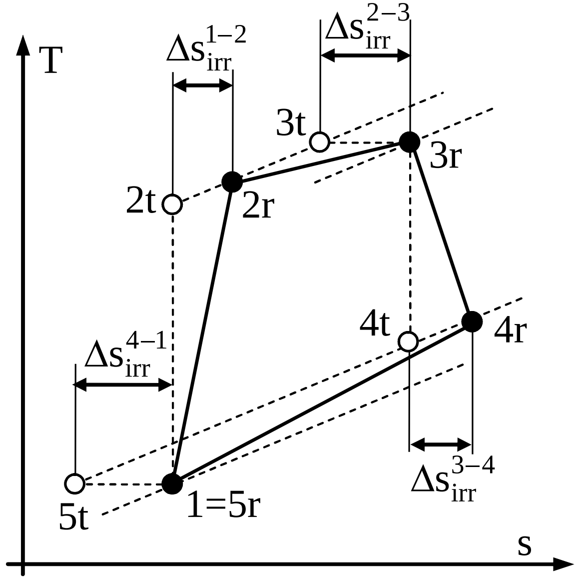
$$\eta_{engine} = \frac{P}{\dot{Q}^{2r-3t}} = 1 - \frac{\left|\dot{Q}_0^{4r-5t}\right|}{\dot{Q}^{2r-3t}} =$$

$$= 1 - \frac{\dot{m} \cdot T_{mq}^{4r-5t} \cdot \left|\Delta s_{q0}\right|}{\dot{m} \cdot T_{mq}^{2r-3t} \cdot \Delta s_q} = 1 - \frac{T_{mq}^{4r-5t}}{T_{mq}^{2r-3t}} N_{irr,int}^{engine}$$

The Irreversible First Law Efficiency

External and Internal Irreversibility

Any Basic Irreversible Engine



$$\varepsilon_{II} = \frac{\dot{Q}^{2r-3t}}{\dot{Q}^{2-3}_{max}} = \frac{\dot{m} T_{mq}^{2r-3t} \Delta s_q}{\dot{m} T_{mq} \Delta s_q} = C_{\Delta s_q} \frac{T_{mq}^{2r-3t}}{T_{mq}} < 1$$

$$\varepsilon_{II,0} = \frac{\dot{Q}^{4r-5t}}{\dot{Q}^{4-5}_{min}} = \frac{\dot{m} T_{mq}^{4r-5t} \Delta s_{q0}}{\dot{m} T_{mq0} \Delta s_{q0}} = C_{\Delta s_{q0}} \frac{T_{mq}^{4r-5t}}{T_{mq0}} > 1$$

$$N_{irr,ext} = \frac{\varepsilon_{II,0}}{\varepsilon_{II}} \frac{C_{\Delta s_q}}{C_{\Delta s_{q0}}} > 1$$

$$\eta_{engine} = 1 - \frac{T_{mq0}}{T_{mq}} \frac{\varepsilon_{II,0}}{\varepsilon_{II}} \frac{C_{\Delta s_q}}{C_{\Delta s_{q0}}} N_{irr,int} = 1 - \frac{1}{\tau} N_{irr,ext} \cdot N_{irr,int}$$

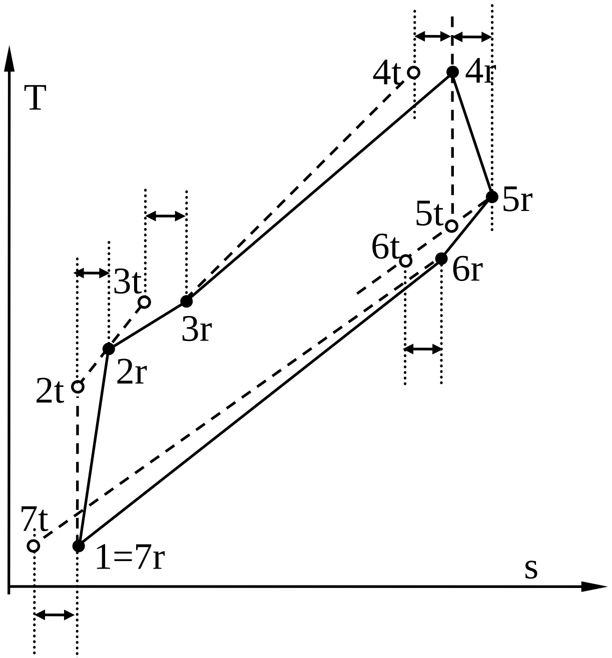
The Irreversible First Law Efficiency

Internal Irreversibility

Any Irreversible Engine with Internal Heat Transfer (Internal Regeneration of Heat)

1 – 2r irreversible adiabatic compression, 2r – 3r internal irreversible heating by heat regeneration, 3r – 4r irreversible heating, 4r – 5r irreversible adiabatic expansion, 5r – 6r internal irreversible cooling by heat regeneration,

6r – 1 irreversible cooling.



$$\Delta s_q = \Delta s_{3-4} = s_{4t} - s_{3r}$$

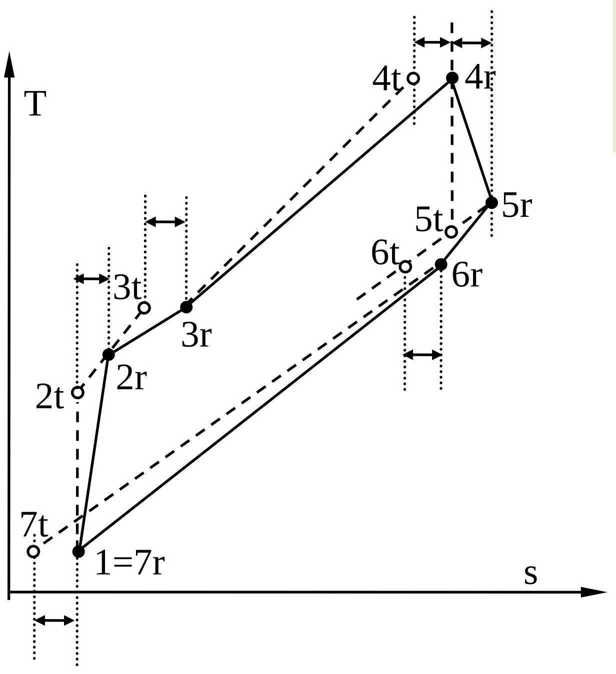
$$T_{mq}^{2r-3t}\left(s_{3t} - s_{2r}\right) = T_{mq}^{5r-6t}\left(s_{5t} - s_{6r}\right)$$

$$\left|\Delta s_{q0}\right| = s_{6r} - s_{7t} = \Delta s_q + \left(s_{3t} - s_{2r}\right)\left(1 - \frac{T_{mq}^{2r-3t}}{T_{mq}^{5r-6t}}\right) + \Delta s_{irr}^{1-2} + \Delta s_{irr}^{2-3} + \Delta s_{irr}^{3-4} + \Delta s_{irr}^{4-5} + \Delta s_{irr}^{5-6} + \Delta s_{irr}^{6-7}$$

The Irreversible First Law Efficiency

Internal Irreversibility

Any Irreversible Engine with Internal Heat Transfer (Internal Regeneration of Heat)



$$N_{irr,int} = \left(1 + \frac{\left(\sum \Delta s_{irr}\right)_{int}}{\Delta s_q} + \frac{\left(s_{3t} - s_{2r}\right)}{\Delta s_q}\left(1 - \frac{T_{mq}^{2r-3t}}{T_{mq}^{5r-6t}}\right)\right) > 1$$

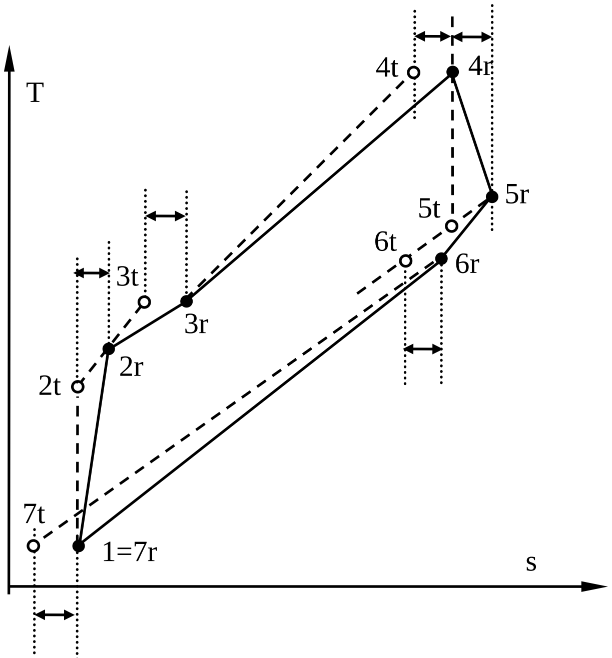
$$\eta_{engine} = \frac{\dot{W}}{\dot{Q}} = 1 - \frac{\left|\dot{Q}_0\right|}{\dot{Q}} = 1 - \frac{\dot{m} \cdot T_{mq}^{6r-7t} \cdot \left|\Delta s_{q0}\right|}{\dot{m} \cdot T_{mq}^{3r-4t} \cdot \Delta s_q} =$$

$$= 1 - \frac{T_{mq}^{6r-7t}}{T_{mq}^{3r-4t}} N_{irr,int}$$

The Irreversible First Law Efficiency

External and Internal Irreversibility

Any Irreversible Engine with Internal Heat Transfer (Internal Regeneration of Heat)



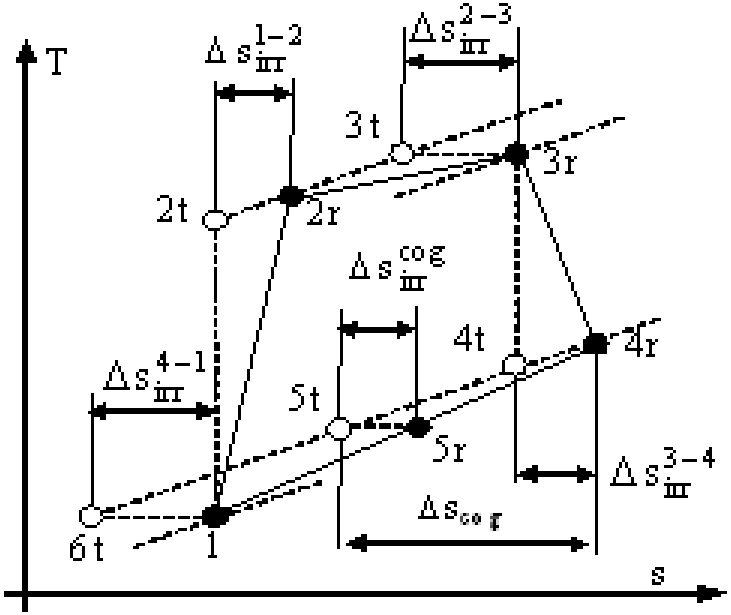
$$\eta_{engine} = 1 - \frac{T_{mq0}}{T_{mq}} \frac{\varepsilon_{II,0}}{\varepsilon_{II}} \frac{C_{\Delta s_q}}{C_{\Delta s_{q0}}} N_{irr,\text{int}} =$$

$$= 1 - \frac{1}{\tau} \cdot N_{irr,ext} \cdot N_{irr,\text{int}}$$

The Irreversible First Law Efficiency

Internal Irreversibility

Any Irreversible Co-generation Engine



$$\Delta s_q = \Delta s_{2-3} = s_{3t} - s_{2r}$$

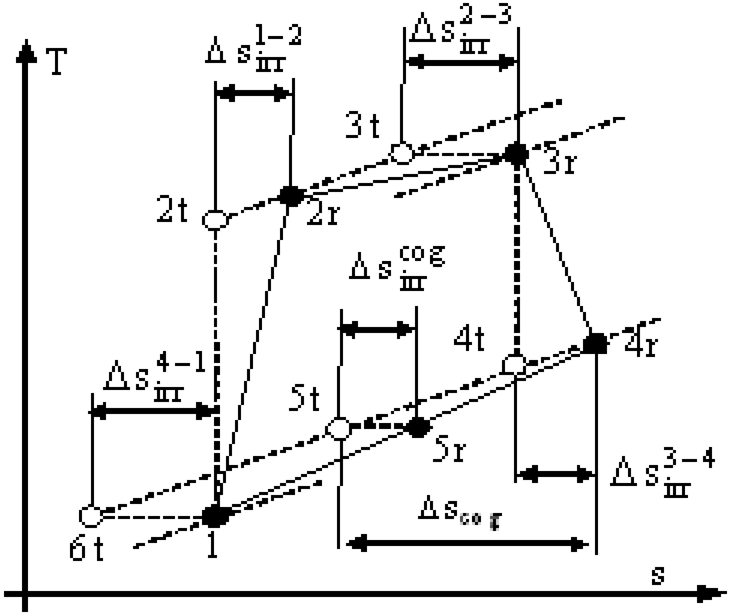
$$|\Delta s_{q0}| = |\Delta s_{4-1}| = s_{4r} - s_{6t} = \Delta s_q + \sum_{x=1,2,3,4}^{y=2,3,4,1} \Delta s_{irr}^{x-y} = \Delta s_q \left(1 + \frac{\left(\sum \Delta s_{irr}\right)_{int}^{engine}}{\Delta s_q} \right)$$

$$\left|\Delta s_{q,0}^{cog}\right| = |\Delta s_{5-1}| = s_{5t} - s_{6t} = |\Delta s_{4-1}| - |\Delta s_{cog}| = \Delta s_q \left(1 + \frac{\left(\sum \Delta s_{irr}\right)_{int}^{engine} - |\Delta s_{cog}|}{\Delta s_q} \right)$$

The Irreversible First Law Efficiency

Internal Irreversibility

Any Irreversible Co-generation Engine



$$\eta_{overall} = \frac{P + \left|\dot{Q}_{4-5}\right|}{\dot{Q}_{2-3}} = 1 - \frac{\left|\dot{Q}_{4-1}\right| - \left|\dot{Q}_{4-5}\right|}{\dot{Q}_{2-3}} =$$

$$1 - \frac{\dot{m}\cdot\left(T_{mq}^{4r-6t}\cdot\left|\Delta s_{4-1}\right| - T_{mq}^{4r-5t}\cdot\left|\Delta s_{4-5}\right|\right)}{\dot{m}\cdot T_{mq}^{2r-3t}\cdot\Delta s_{2-3}} =$$

$$= 1 - \frac{T_{mq}^{4r-6t}}{T_{mq}^{2r-3t}} N_{irr,int}^{cog}$$

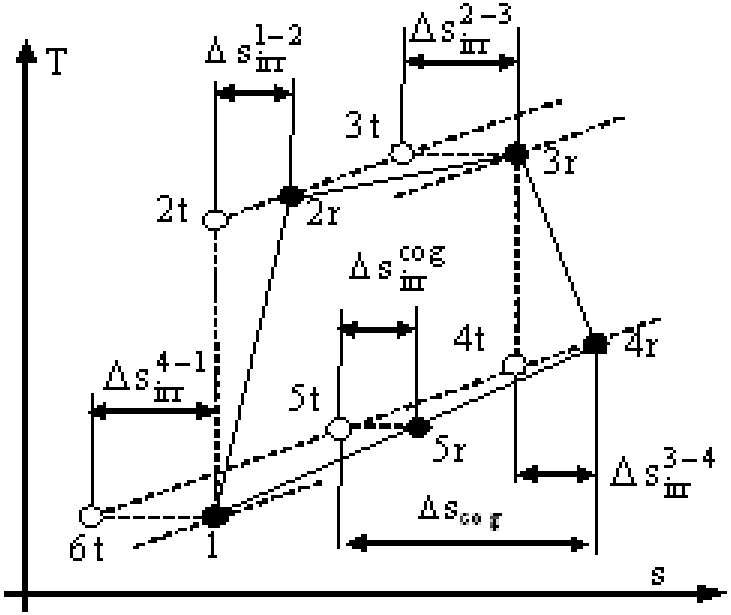
$$N_{irr,int}^{cog} = \left(1 + \frac{\left(\sum \Delta s_{irr}\right)_{int}^{engine}}{\Delta s_{2-3}} - \frac{T_{mq}^{4r-5t}}{T_{mq}^{4r-6t}}\frac{\Delta s_{cog}}{\Delta s_{2-3}}\right)$$

For endo-reversibility $N_{irr,int}^{cog} \to 0$

The Irreversible First Law Efficiency

External and Internal Irreversibility

Any Irreversible Co-generation Engine



$$\eta_{overall} = 1 - \frac{T_{mq0}}{T_{mq}} \frac{\varepsilon_{II,0}}{\varepsilon_{II}} \frac{C_{\Delta s_q}}{C_{\Delta s_{q0}}} N_{irr,int} =$$

$$= 1 - \frac{1}{\tau} N_{irr,ext} N_{irr,int}$$

For endo-reversibility $N_{irr,int}^{cog} \rightarrow 0$ and $\eta_{overall} \rightarrow 1$

Numerical Results

The computational procedure assumed the following restrictive conditions.

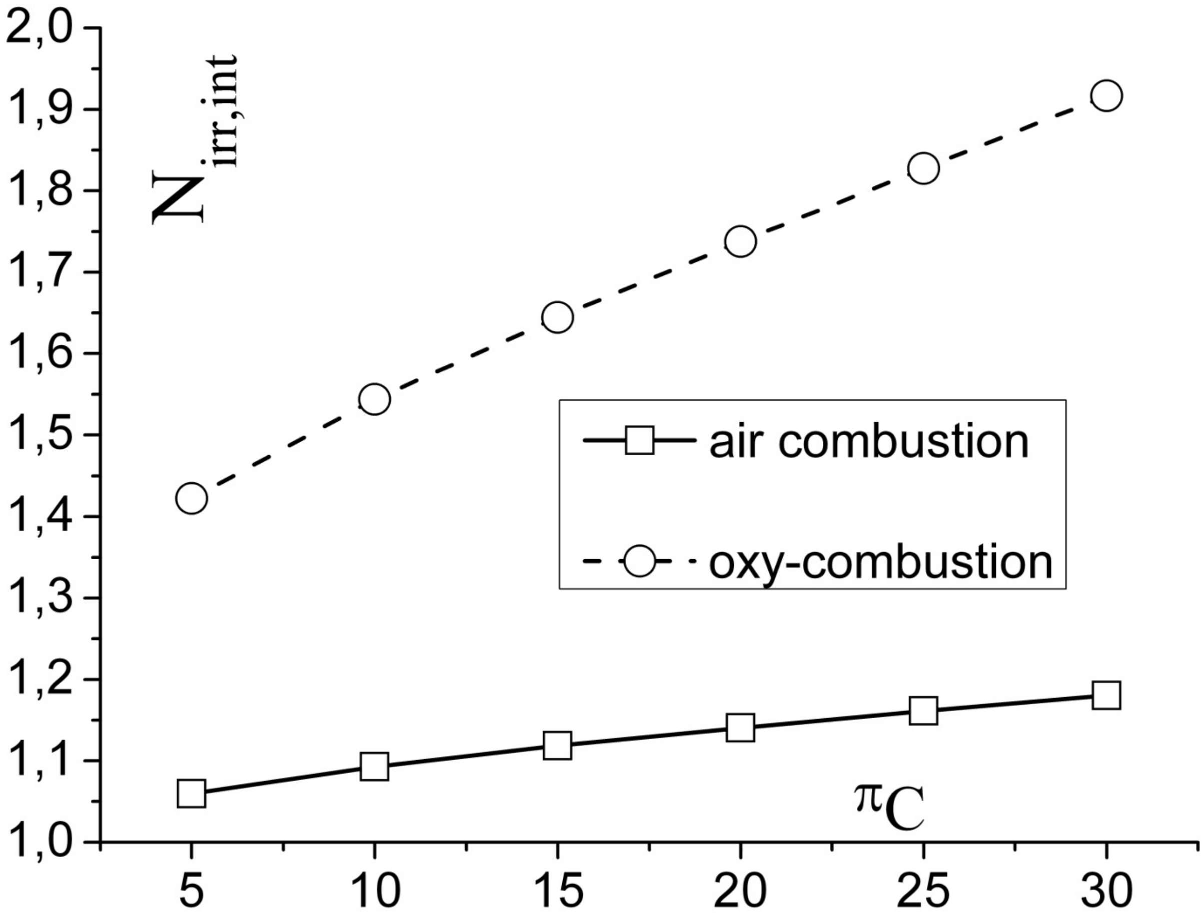
- *Variable heat capacities of gases, fourth degree temperature polynomials;*
- *Adiabatic exponents of the reversible processes computed as the ratio of enthalpy variation to internal energy variation.*
- *In the case of engine heated by combustion, they were considered negligible dissociation during combustion, and the mass and energy balance equations of combustion gave the flue gases composition, the excess air value, and the fuel mass flow rate.*

Numerical Results

First Figures compare the oxy-combustion with 90% mass fraction of O_2 in the oxygenated air, and the air combustion, for an open Joule – Brayton cogeneration cycle. The restrictive conditions were: maximum temperature on the cycle, 1200°C; isentropic efficiency of the compressor, 0.88; isentropic efficiency of the gas turbine, 0.94; pressure loss coefficient in the combustion chamber, 0.98; the fuel mass composition of 15% H_2 and 85% C; higher heating value of the fuel ~46,000 kJ/kg.

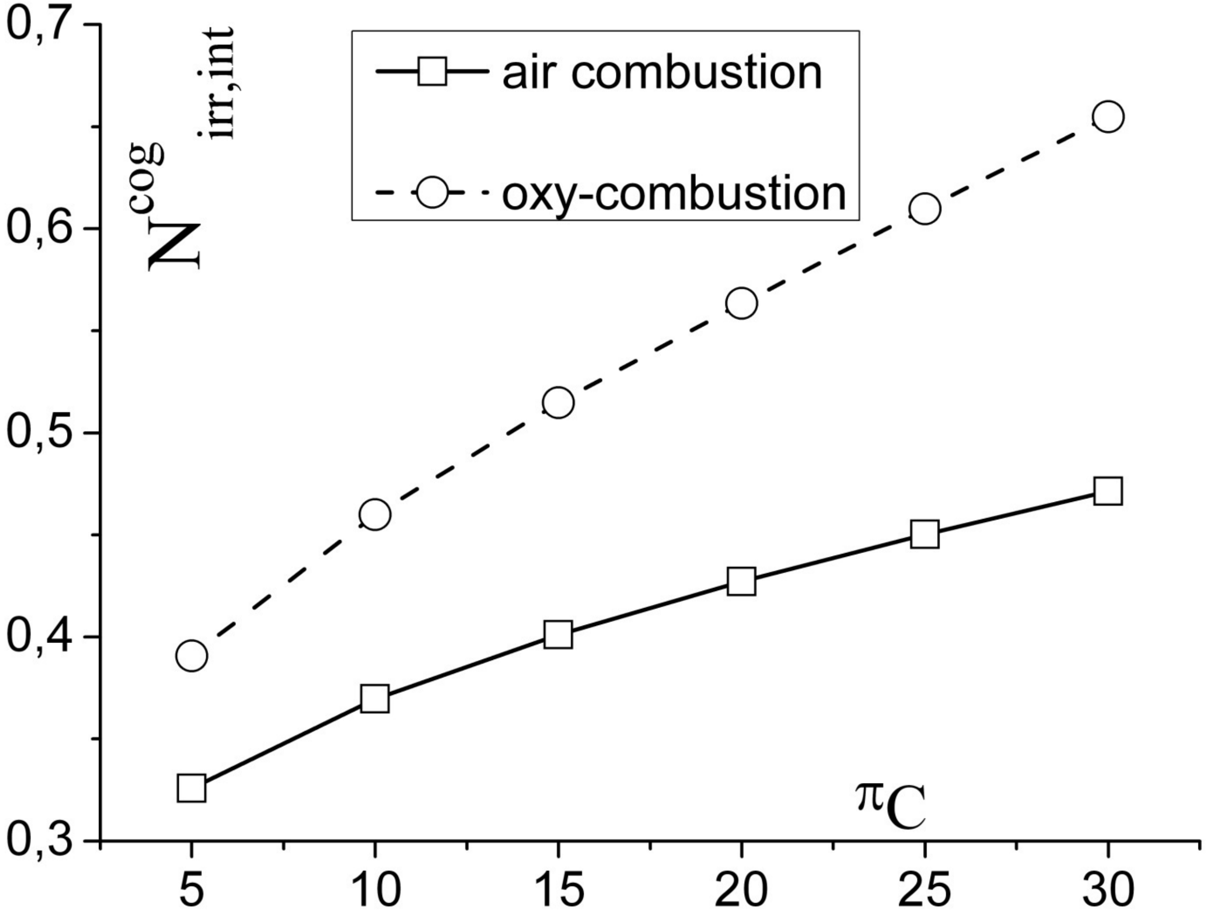
Next Figures compare six possible working fluids, air, O_2, N_2, CO_2, H_2 and He, in a closed Joule – Brayton engine cycle externally heated, e.g. by solar energy. The restrictive conditions were: maximum temperature on the cycle, 1000°C; the minimum temperature on the cycle, 20°C; isentropic efficiency of the compressor, 0.88; isentropic efficiency of the gas turbine, 0.94; pressure loss coefficient in the heat exchanger connecting the external hot source, 0.98; pressure loss coefficient in the heat exchanger connecting the external cold sink, 0.98; variable heat capacities; adiabatic exponent computed as the ratio of heat capacities at constant pressure and constant volume.

Numerical Results



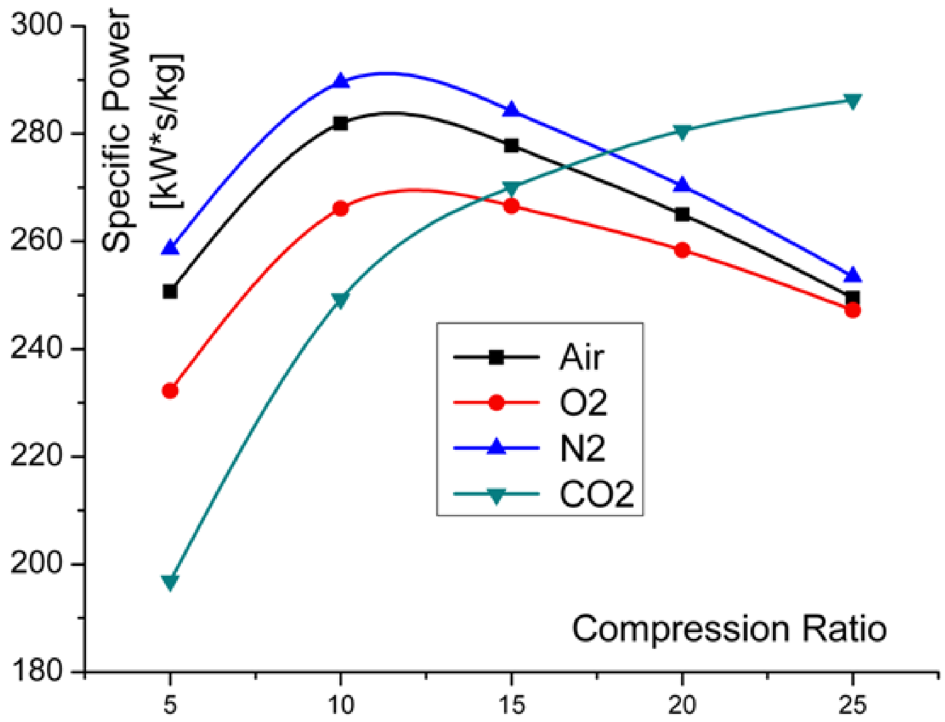
The dependence number of internal irreversibility – compression ratio for the basic engine cycle

Numerical Results



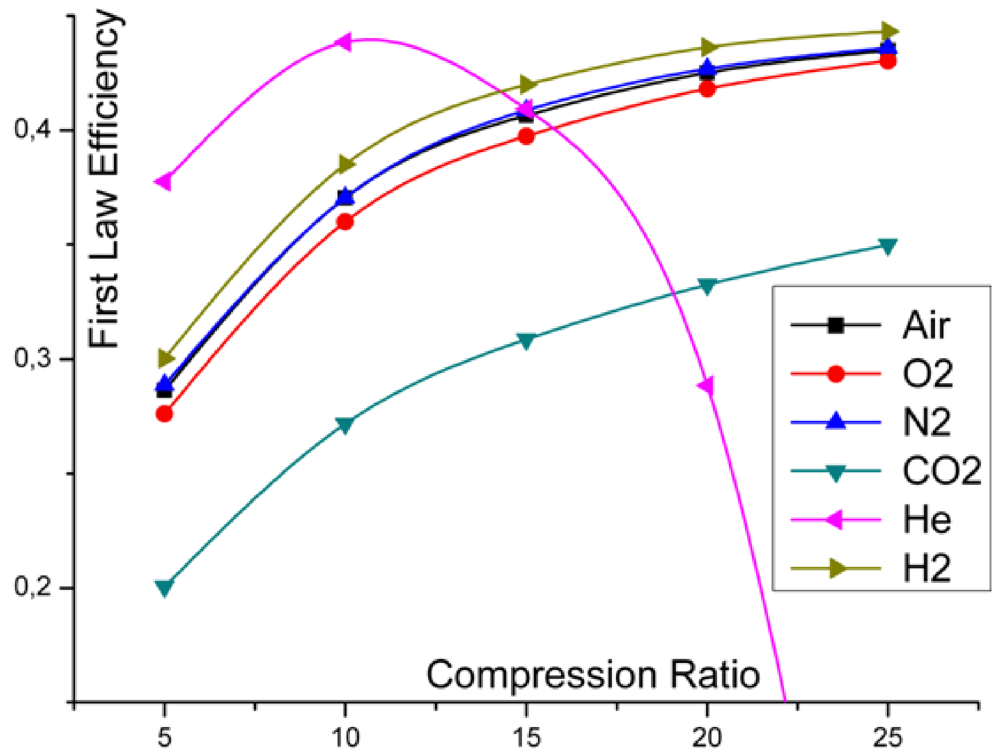
The dependence overall number of internal irreversibility – compression ratio for the cogeneration cycle

Numerical Results



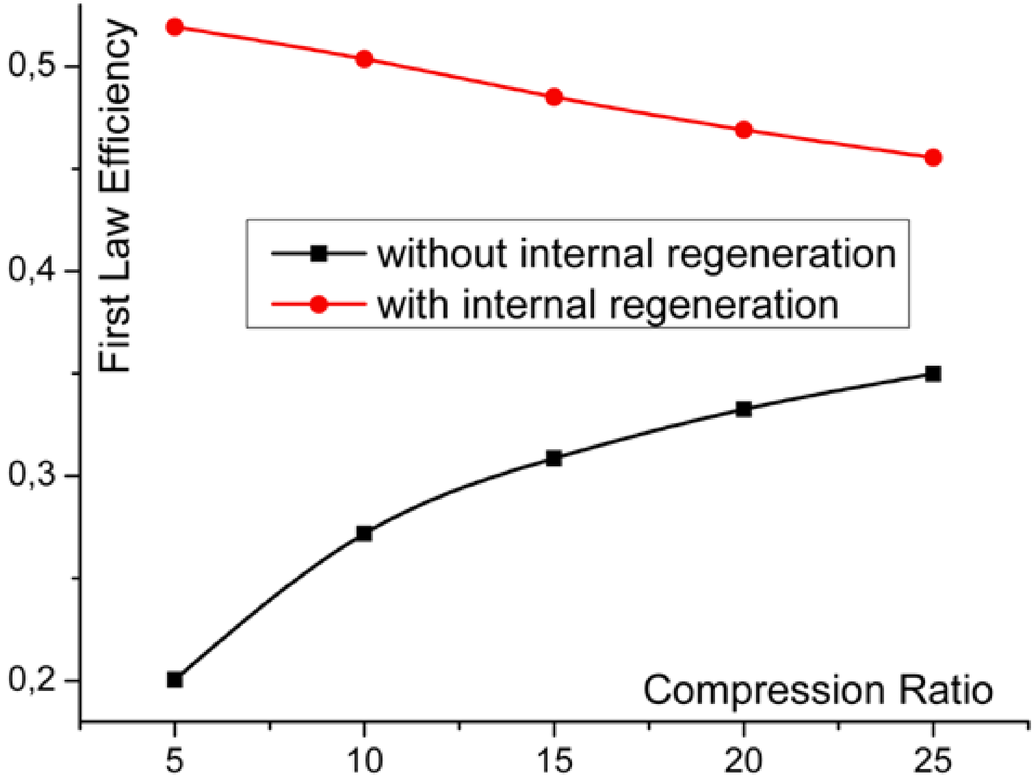
The dependence specific power output on compression ratio for the basic engine cycle

Numerical Results



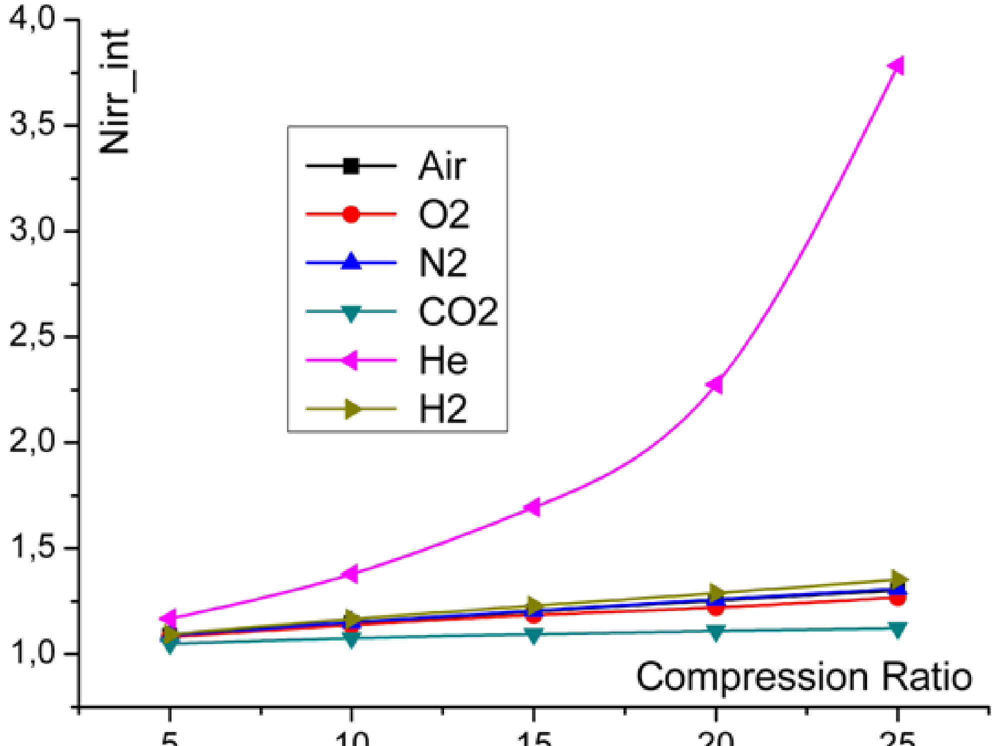
The dependence irreversible first law efficiency (thermodynamic efficiency) on compression ratio for the basic engine cycle

Numerical Results



Comparison of the dependence irreversible first law efficiency (thermodynamic efficiency) on compression ratio for both the basic engine cycle and the engine cycle with internal regeneration of heat, only for CO_2 as working fluid

Numerical Results



The dependence number of internal irreversibility on compression ratio for the basic engine cycle

Conclusions

This paper deals with a new optimization criteria, the irreversible first law efficiency applied to the real cycles.

This new approach emphasizes the overall irreversibility by the means of the numbers of internal and external irreversibility, directly inside the first law efficiency.

It yields that the maximum power is not unique, as it was assumed untill now, respectively depends on the nature of the working fluid and on the restrictive conditions.

The irreversible first law efficiency conducts to the reversible limit, i.e. the reversible Carnot engine, respectively offers the simplest comparison.

Conclusions

For complex cycles, we have only to deal carefully with the entropy variations and thermal interactions during the heat transfer processes.

Important remark: *the second law effectiveness, defined in this paper, takes into account only the thermal interactions connecting the working fluids with external heat reservoirs, the internal heat exchanges of complex cycles, e.g. in the case of combined cycles, the heat transfer between the top cycle and the bottom one is included in the number of internal irreversibility; for instance, refer to the simple manner to manage the internal heat exchange of Power Cycles with Internal Heat Transfer.*

Conclusions

The direct method presented in this presentation is a concise one, equivalent to exergy analysis, since the lost exergy is proportional to the entropy irreversibly generated. The comparison of possible fluids used in a closed Joule – Brayton cycle, externally heated, shows that CO_2 offers a substantial potential for internal regeneration of heat, see the below table, that includes the difference of temperatures of the fluid leaving the turbine and the compressor.

Conclusions

Table 1. The difference between temperatures of the fluid leaving the turbine and the compressor, [℃]

Compression Ratio	Air	O_2	N_2	CO_2	He	H_2
5,00	**401**	**423**	**397**	**593**	**123**	**367**
10,00	**158**	**191**	**152**	**431**	41	**115**
15,00	17	57	9	**337**		
20,00				**272**		
25,00				**221**		

The Influences of the Condenser and Evaporator Effectiveness on the Heat Pump Performances

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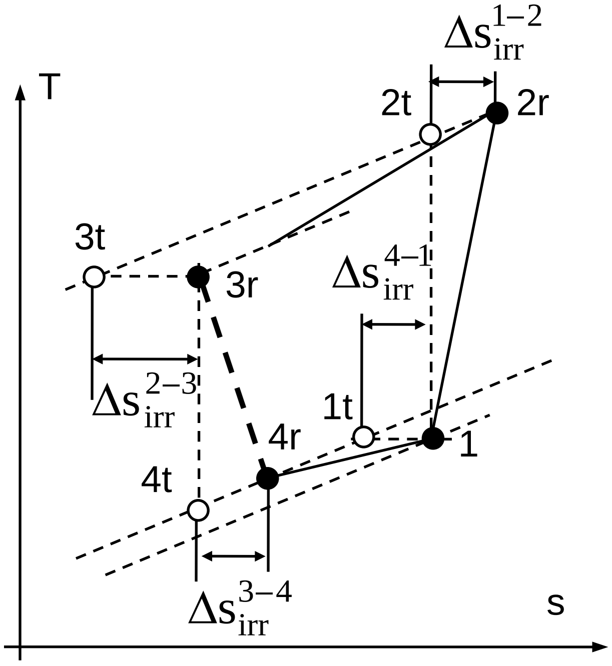
C.N.R.S. UMR 7563

Overview

1. ***Introduction***
2. ***Mathematical model***
3. ***Results and discussion***
4. ***Conclusions***

1. Introduction

First and Second Laws Relationship – CV-HP



- ***First Law – Energy Balance***

$$\dot{Q} = Q_e = \dot{Q}_{4r-1t} = \dot{m}T_{mq}^{4r-1t}\left(s_{1t} - s_{4r}\right) = \dot{m}T_{mq}^{4r-1t}\Delta s_q > 0$$

$$\dot{Q}_0 = \dot{Q}_c = \dot{Q}_{2r-3t} = \dot{m}T_{mq}^{2r-3t}\left(s_{3t} - s_{2r}\right) = \dot{m}T_{mq}^{2r-3t}\Delta s_{q0} < 0$$

$$\dot{W} = \dot{Q} + \dot{Q}_0 = \dot{Q} - \left|\dot{Q}_0\right| < 0$$

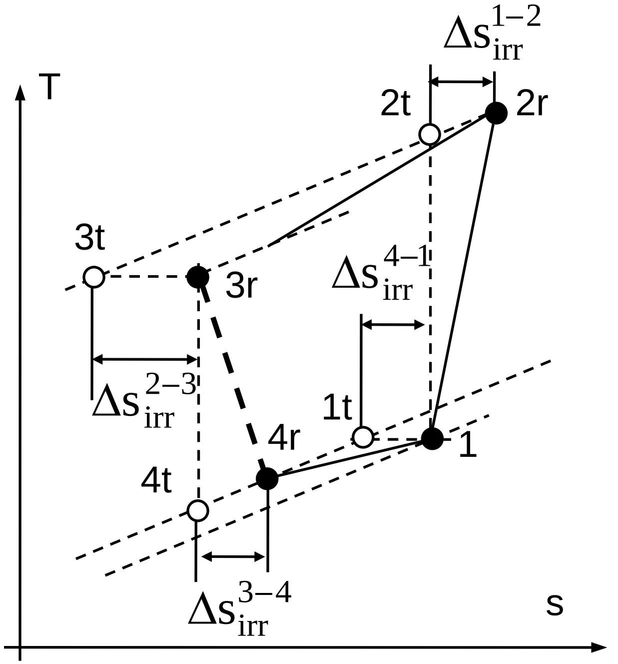
$$\Delta s_q = s_{1t} - s_{4r}$$

$$\Delta s_{q0} = s_{3t} - s_{2r} = \Delta s_q + \Delta s_{irrev}^{4-1} + \Delta s_{irrev}^{1-2} + \Delta s_{irrev}^{2-3} + \Delta s_{irrev}^{3-4}$$

$$= \Delta s_q + \left(\sum \Delta s_{irrev}\right)_{int}$$

1. Introduction

First and Second Laws Relationship – CV-HP



- ***First Law – Energy Balance***

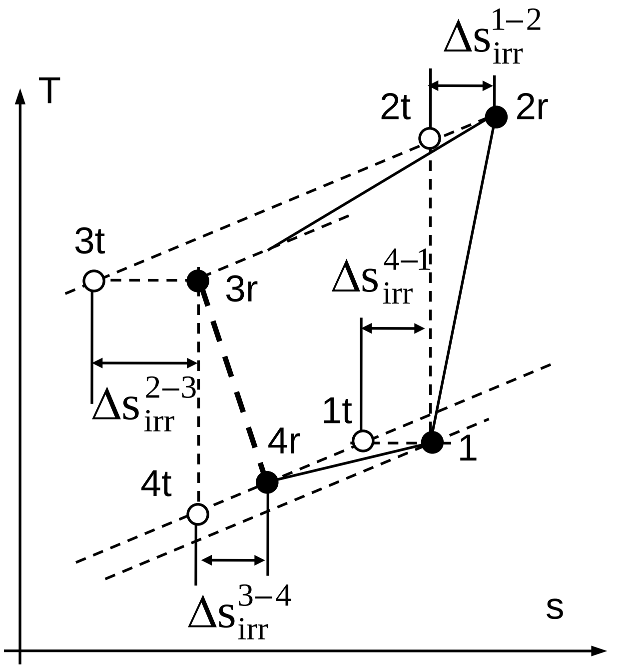
$$COP_{HP}=\frac{\left|\dot{Q}_0\right|}{\left|\dot{Q}_0\right|-\dot{Q}}=\frac{\dot{m}T_{mq}^{2r-3t}\left|\Delta s_{q0}\right|}{\dot{m}\left(T_{mq}^{2r-3t}\left|\Delta s_{q0}\right|-T_{mq}^{4r-1t}\Delta s_q\right)}=$$

$$=\frac{\dfrac{\left|\dot{Q}_0\right|}{\dot{Q}}}{\dfrac{\left|\dot{Q}_0\right|}{\dot{Q}}-1}=\frac{\dfrac{T_{mq}^{2r-3t}}{T_{mq}^{4r-1t}}N_{irr,int}}{\dfrac{T_{mq}^{2r-3t}}{T_{mq}^{4r-1t}}N_{irr,int}-1}\underset{HP}{\overset{e.g.}{=}}\frac{\dfrac{T_c}{T_e}Irr}{\dfrac{T_c}{T_e}Irr-1}$$

$$N_{int,irr}=1+\frac{\left(\sum\Delta s_{irr}\right)_{int}}{\Delta s_q}>1\ \text{or,}\ \ Irr=N_{int,irr}\frac{T_{mq}^{2r-3t}}{T_{ref}^{2-3}}\frac{T_e}{T_c}$$

1. Introduction

First and Second Laws Relationship – CV-HP



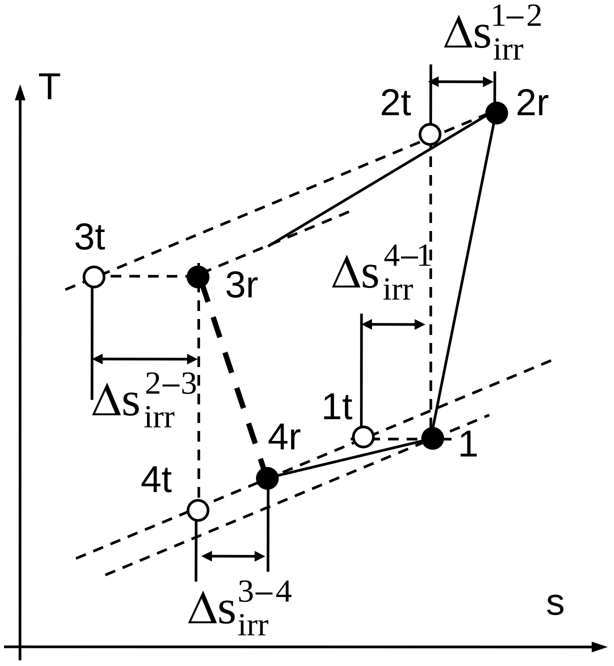
- ***Second Law – entropy balance***
  - ***Internal irreversibility***

$$\frac{\dot{Q}}{T_{mq}^{4r-1t}} N_{irr,int} - \frac{|\dot{Q}_0|}{T_{mq}^{2r-3t}} = 0 \quad \text{or} \quad \frac{\dot{Q}}{T_{ref}^{4-1}} \text{Irr} - \frac{|\dot{Q}_0|}{T_{ref}^{2-3}} = 0$$

$$\text{or e.g. (CV - HP)} \quad \frac{\dot{Q}_e}{T_e} \text{Irr} - \frac{|\dot{Q}_c|}{T_c} = 0$$

1. Introduction

First and Second Laws Relationship – CV-HP



- ***United First and Second Laws***

$$N_{irr,int} = COPR\,\frac{T_{mq}^{4r-1t}}{T_{mq}^{2r-3t}} \quad \text{or} \quad Irr = COPR\,\frac{T_{ref}^{4-1}}{T_{ref}^{2-3}}$$

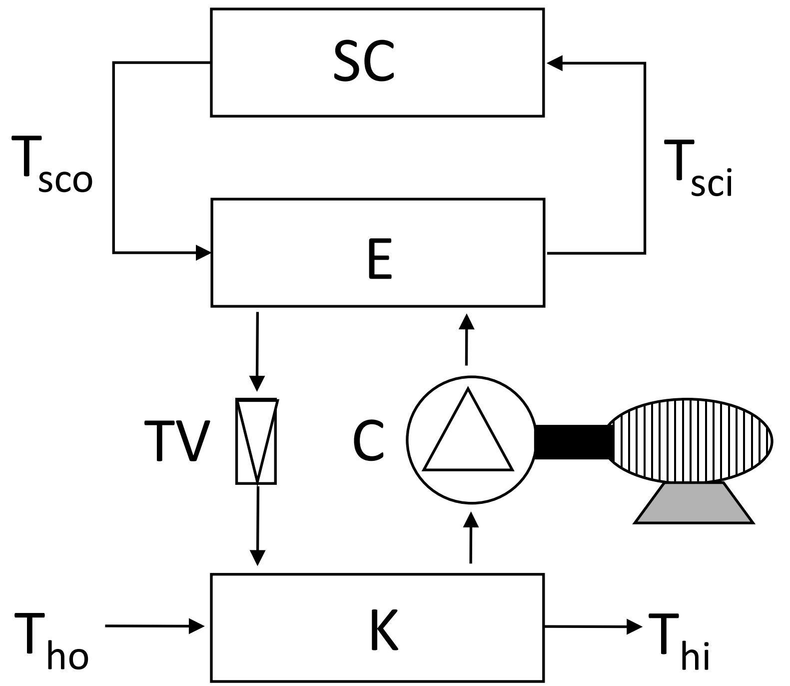
$$\text{or e.g. (CV - HP) } Irr = COPR\,\frac{T_e}{T_c}$$

$$\text{where } COPR = \frac{COP_{HP}}{COP_{HP} - 1}$$

2. Mathematical model

Thermal Relationships of a Solar Assisted Compression

Vapor Heat Pump (CV-HP)



2. Mathematical model

- ***Solar collector***
  - Collector temperature under non-flow conditions $$T_s = T_a + \frac{\eta_o I}{U_L}$$
  - Collector plate temperature $$T_{sc} = T_s - \frac{\eta_s I}{U_L}$$
  - Inlet temperature of the solar heat carrier $$T_{sci} = T_{sc} - \frac{\eta_s IA}{\varepsilon_{sc} \dot{C}_{sc}}$$
  - Outlet temperature of the solar heat carrier $$T_{sco} = T_{sci} + \frac{\eta_s IA}{\dot{C}_{sc}}$$

η_o is optical efficiency of solar collector, Ta is environmental temperature, U_L is solar collector heat-loss coefficient; ε_{sc} is solar collector effectiveness; *I is total solar radiation*.

2. Mathematical model

- ***Heating system***

Inlet/outlet temperatures of the useful heat carrier, were imposed by considering that the heating system asks:

✓ at $T_a = T_{aN}$ = 253.15K, $T_{hi} = T_{hiN}$ = 323.15K and, $T_{ho} = T_{hoN}$ = 333.15K;

✓ at $T_a = T_{aS}$ = 288.15K, $T_{hi} = T_{hiS}$ = 303.15K;

✓ for 253.15 K < T_a < 288.15 K: $T_{ho} \cong -0.5714286 \cdot T_a + 467.807143$

✓ $T_{room} = 295.15K$.

$$\frac{\dot{Q}_{heating}(T_a)}{\dot{Q}_{heating}(T_{aN})} = \frac{T_{hi} - T_{ho}}{T_{hiN} - T_{hoN}} \cong \frac{T_{room} - T_a}{T_{room} - T_{aN}} \Rightarrow T_{hi}(T_a)$$

2. Mathematical model

- ***Evaporator***

evaporator heat rate $$\dot{Q}_e = \dot{C}_{sc}\left(T_{sco} - T_{sci}\right)$$

evaporator temperature $$T_e = T_{sco} - \frac{\dot{Q}_e}{\varepsilon_e \dot{C}_{sc}}$$

Where ε_e is evaporator effectiveness.

2. Mathematical model

- ***Heat pump irreversibility*** *(entropy and energy balance)*

$$\mathrm{Irr}\frac{\dot{Q}_e}{T_e} - \frac{\left|\dot{Q}_c\right|}{T_c} = 0 \;\Rightarrow\; \dot{Q}_c$$

$$\mathrm{Irr} = \frac{\left(\mathrm{COP_{HP}} + 1\right)}{\mathrm{COP_{HP}}}\frac{T_e}{T_c} = \mathrm{COPR}\,\frac{T_e}{T_c}$$

2. Mathematical model

- ***Condenser***

Condensing temperature

$$T_c = T_{ho} + \frac{\left|\dot{Q}_c\right|}{\varepsilon_c \dot{C}_h}$$

Where ε_c *is condenser effectiveness*

2. Mathematical model

By CoolPack, and for adopted restrictive conditions inside the heat pump, it was interpolated, by square roots, the irreversibility, for $303.15K < T_c < 338.15K$, and, $258.15K < T_e < 288.15K$:

- refrigerant R22, errors: +1.1%, – 3.1% $$\mathrm{COPR} = -0.90971003 + 1.850627\frac{T_c}{T_e}$$
- refrigerant R717, errors: +1.1%, – 1.8% $$\mathrm{COPR} = -0.60229 + 1.568338\frac{T_c}{T_e}$$
- refrigerant R410A, errors: +1.1%, – 6.6% $$\mathrm{COPR} = -1.68326472 + 2.551254\frac{T_c}{T_e}$$
- refrigerant R407c, errors: +1.1%, – 4.9% $$\mathrm{COPR} = -1.371045642 + 2.272844431\frac{T_c}{T_e}$$
- refrigerant R290, errors: +1.1%, – 3.7% $$\mathrm{COPR} = -1.02348136 + 1.951886275\frac{T_c}{T_e}$$
- refrigerant R134a, errors: +1.1%, – 3.6% $$\mathrm{COPR} = -0.9796944 + 1.9103469052\frac{T_c}{T_e}$$

2. Mathematical model

- ***Power balance***

$$\dot{W} = \dot{Q}_e + \dot{Q}_c = \dot{Q}_e - \left|\dot{Q}_c\right|$$

$$COP_{HP} = \frac{\left|\dot{Q}_c\right|}{\left|\dot{W}\right|}$$

3. Results and discussion

- The correlated compilation of previous equations allowed in MathLab to find out numerical results, depending only on ε_e and ε_c. Figures 2 to 5 show selected numerical results for:

$$T_a = 273.15K; I = 50\frac{W}{m^2}; \eta_o = 0.95; U_L = 2\frac{W}{m^2 K};$$

$$\dot{C}_{sc} = 1000\frac{J}{KgK}; \dot{C}_h = 1000\frac{J}{KgK}; \eta_s = 0.75; \varepsilon_{sc} = 0.75$$

3. Results and discussion

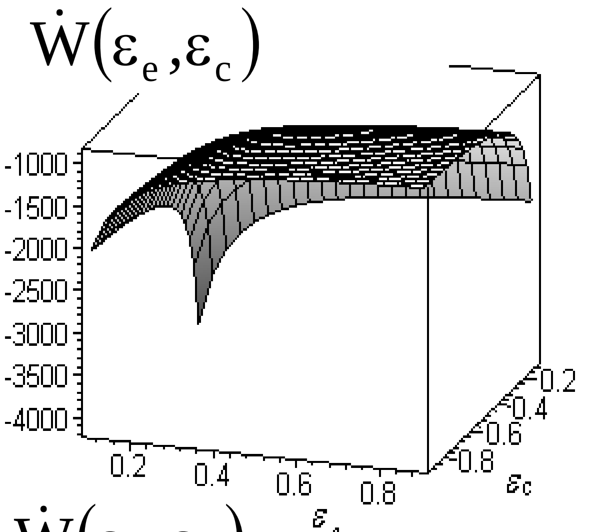


Fig.2. R22

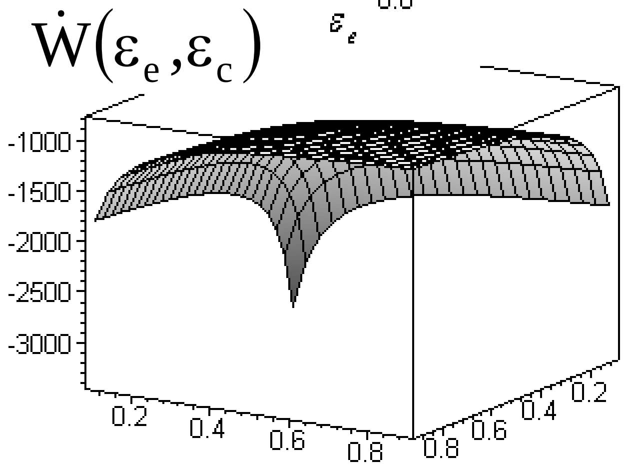


Fig.3. R717

3. Results and discussion

$COP_{HP}(\varepsilon_e, \varepsilon_c)$

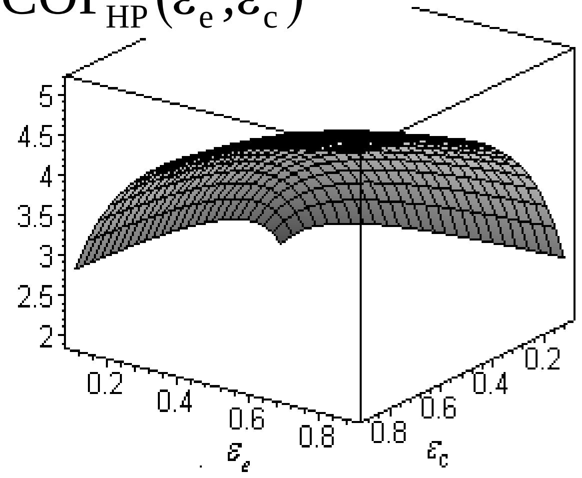


Fig.4. R22

$COP_{HP}(\varepsilon_e, \varepsilon_c)$

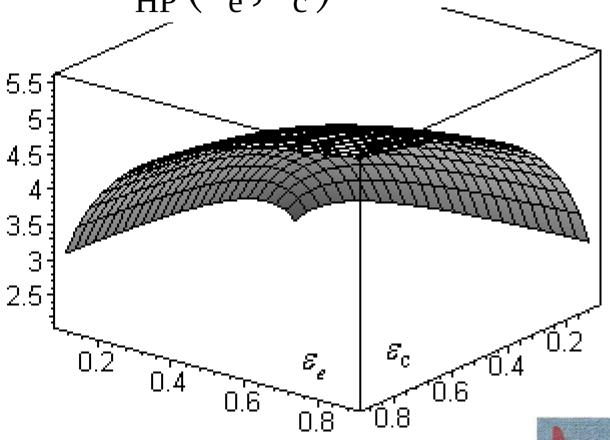


Fig.5. R717

3. Conclusions

The presentation presents a mathematical algorithm able in modeling the influences of evaporator and condenser effectiveness upon the all operational features of a heat pump: power, heat rates, and temperatures. The model start with the external heat sources imposed parameters and, combine these ones by the internal irreversibility of the heat pump (entropy balance).